A Superfluid Universe

Lecture 3
The big bang & quantum turbulence

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Lecture 3.

• Scale change & renormalization
  RG trajectories, fixed points
  Halpern-Huang scalar field

• Cosmological equations

• Planck scale and nuclear scale

• Quantum turbulence – inflation era

• Dark energy

• Dark mass, galactic voids, and other phenomena
Big bang and renormalization

• Big bang is characterized by rapid change of length scale.

• Therefore, the cutoff of quantum fields change rapidly.

• With change of cutoff, interactions undergo renormalization.

\[
\mathcal{L} = \partial \phi^* \partial \phi - V(\phi^* \phi)
\]

\[
V(\phi^* \phi) = \lambda_2 \phi^* \phi + \lambda_4 (\phi^* \phi)^2 + \lambda_6 (\phi^* \phi)^3 + \cdots
\]

All the coupling constants change rapidly during the big bang.
In quantum field theory a cutoff is needed to suppress high-frequency virtual processes.

- The cutoff is the only scale in the theory
  \[ \Lambda_0 \rightarrow \Lambda \]
- Scale change
- All coupling constants undergo renormalization to preserve the identity of the theory.
- The “appearance” of the theory is changed, but not the identity.

Scale changes form a group. Operations of renormalization form a representation called the renormalization group (RG).
RG trajectory

\[ \mathcal{L} = \partial^* \partial + V(\phi^* \phi, \Lambda) \]

As \( \Lambda \) changes, \( V(\phi^* \phi, \Lambda) \) traces out an RG trajectory in the space of Lagrangians.

\[ \rightarrow \text{coarse-graining direction} \]
Renormalization

Appearances change under scale change, but not intrinsic identity.
RG trajectories

- Fixed point
- UV trajectory
  Asymptotic freedom
  QCD
- IR trajectory
  Non-free
  QED, $\phi^4$
- Gaussian fixed point:
  Free massless scalar field
The Creation

• At big bang, universe at **Gaussian fixed point**: \( \Lambda = \infty, V \equiv 0 \)

• Scalar field gets on RG trajectory along particular direction (at random?)

• Direction corresponds particular form of \( V \).

Outgoing trajectory --- Asymptotic freedom Potential grows to spawn a possible universe.

Ingoing trajectory --- Triviality Universe never left the fixed point.
Radius of universe gives field-theory cutoff:

\[ \Lambda = \frac{\hbar}{a} \]

All asymptotically free potentials are **Halpern-Huang** potentials:

\[ V = c\Lambda^{4-b} \left[ M\left( -2 + \frac{b}{2}, \frac{N}{2}, z \right) - 1 \right] \quad (0 < b < 2) \]

\[ z = \frac{8\pi^2|\phi|^2}{\Lambda^2} \]

\( M \) = Kummer function

\( N \) = No. of field components

For large fields:

\[ M(p, q, z) \approx \Gamma(q) \Gamma^{-1}(p) z^{p-q} \exp z \]

The field theory is \( D=4 \) generalization of the sine-Gordon theory in \( D=2 \).
The big bang

Sub-Planckian era

- Universe very hot
- Rapidly cools
- Bose-Einstein condensation

Model starts

- Robertson-Walker metric
- Uniform complex vacuum field, HH potential
- Homogeneous quantum turbulence

Matter created

- Model ceases to be valid.
- Density fluc. Important,
- Standard hot big bang model takes over, but with superfluidity.
Relevant scales

Planck scale: Planck energy $= 10^{18}$ GeV
(Built into Einstein’s equation)

Nuclear scale: Nuclear energy $= 1$ GeV
This scale emerges spontaneously in QCD,
through formation of nucleon bound state
(“dimensional transmutation”).

Example of dimensional transmutation:

2D Schrödinger equation

$$-\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi - \lambda\delta(x)\delta(y) = E\psi \quad (\lambda > 0)$$

$\lambda$ is dimensionless.
There is no intrinsic scale.
But there always exists a bound state.
Its energy is an emergent scale.
Robertson-Walker metric:

\[ ds^2 = -dt^2 + a^2(t)\left( \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (k = 0, 1, -1) \]

For HH complex scalar field:

\[ 
\rho = \frac{1}{2} |\dot{\phi}|^2 + V \\
\rho = \frac{1}{2} |\dot{\phi}|^2 - V - \frac{a}{3} \frac{\partial V}{\partial a} \quad \text{(Last term = trace anomaly)}
\]

FLRW equations of motion: \( H = \dot{a} / a \)

\[ 
\dot{H} = \frac{k}{a^2} - (p + \rho) \\
H^2 = -\frac{k}{a^2} + \frac{2}{3} \rho \\
\dot{\rho} = 3H(\rho + p)
\]

Plus scalar-field and matter equations
• In order to use the Robertson-Walker metric, the scalar field must be spatially uniform.

• To describe vorticity in this metric, take vortex lines to be tubes, outside of which field is uniform.

Space becomes multiply-connected.
**Variables**

\[ a(t) = \text{Radius of universe} \]
\[ F(t) = \text{Modulus of scalar field} \]
\[ \ell(t) = \text{Vortex tube density} \quad E_v = a^3 \varepsilon_0 \ell \quad \text{(Total vortex energy)} \]
\[ \rho(t) = \text{Matter density} \quad E_m = a^3 \rho \quad \text{(Total matter energy)} \]

**Dynamics:**

\[ \dot{a}(t) \quad \text{from Einstein's equation with RW metric.} \]
Source of gravity: \[ T_{\text{tot}}^{\mu\nu} = T_F^{\mu\nu} + T_{\ell}^{\mu\nu} + T_{\rho}^{\mu\nu} \]

\[ \dot{F}(t) \quad \text{from field equation.} \]

\[ \dot{\ell}(t) \quad \text{from Vinen's equation.} \]

\[ \dot{\rho}(t) \quad \text{determined by conservation law } T_{\text{tot}; \mu}^{\mu} = 0. \]
Cosmological equations: \( 4\pi G = c = \hbar = 1 \)

\[ H = \frac{\dot{a}}{a} \]

Planck scale

\[
\frac{dH}{dt} = \frac{k}{a^2} - 2\left(\frac{dF}{dt}\right)^2 + \frac{a}{3} \frac{\partial V}{\partial a} - \frac{1}{a^3} \left( E_m + E_v \right)
\]

\[
\frac{d^2F}{dt^2} = -3H \frac{dF}{dt} - \frac{\zeta_0 E_v}{a^3} F - \frac{1}{2} \frac{\partial V}{\partial F}
\]

Nuclear scale

\[
\frac{dE_v}{d\tau} = -E_v^2 + \gamma E_v^{3/2}
\]

\[
\frac{dE_m}{d\tau} = \left( \frac{\zeta_0}{s_1} \frac{dF^2}{dt} \right) E_v
\]

Constraint:

\[
H^2 + \frac{k}{a^2} - \frac{2}{3} \left( \dot{F}^2 + V + \frac{1+\zeta_0}{a^3} E_v + \frac{1}{a^3} E_m \right) = 0
\]

The two sets decouple because

\[
S_1 = \frac{\tau}{t} = \frac{\text{Planck time scale}}{\text{Nuclear time scale}} = \frac{\text{Nuclear energy scale}}{\text{Planck energy scale}} \approx 10^{-18}
\]
Numerical solutions

\[ H \approx t^{-p} \]
\[ a \approx \exp \left( t^{1-p} \right) \]

- Time-averaged asymptotic behavior: power law
- Gives dark energy without “fine-tuning” problem
Inflation era: quantum turbulence

Radius increases by factor $10^{27}$
- in $10^{-26}$ seconds.
- Matter created = $10^{22}$ sun masses
- Eventually form galaxies outside of vortex cores.

At end of quantum turbulence, model goes over to standard hot big bang theory. But universe remains a superfluid with vorticity.
Inflation era = lifetime of quantum turbulence

• Vortex tangle (quantum turbulence) grows and decays.

• All the matter needed for galaxy formation was created in the tangle.

Legacy

• After decay of quantum turbulence, standard hot big bang theory takes over.

• But the universe remains a superfluid.
Dark energy

Galactic redshift \( (d_L = \text{luminosity distance}) \)

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Hubble’s law: horizontal line
Dark energy: accelerated expansion (points lie above line)

Theory: A,B.
Exponents \( p \) only affects vertical displacement.
The “stick man” simulated by three vortex cores. Galaxies stick to surface of vortex tube.
• Slice cylinder of data into stacks.
• Compare voids in successive stacks.
• Do they form a tube?
Dark mass and “non-thermal filaments”

• Superfluid can be pinned by a random potential

• Galaxy or star cluster could drag the superfluid with it in rotation, acquire extra moment of inertia seen as dark mass.

• Between the co-rotating superfluid and the background will be a boundary layer laced with vortex lines, manifested as the “non-thermal filaments”.

“Non-thermal filaments” observed near the center of the Milky Way
In later universe, reconnections of huge vortex tubes will be rare but spectacular.

Gamma ray burst

- A few events per galaxy per million years
- Lasting ms to minutes
- Energy output in 1 s = Sun’s output in entire life (billions of years).

Cosmic jet

Jet of matter 27 light years long