**Title:** Finite element methods for non H1 space very weak solutions. Part I. Multiscale and stabilization for convection-dominated problems

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**Abstract:** In this talk I will report my recent work on multiscale stabilization finite element method for convection-dominated problem. The solution of this type of problem exhibits boundary-and/or interior layer effects in the classical finite element method. In other words, when there are a finite number of subregions in which the $H^1$ semi-norm of the solution is large but the $H^2$ semi-norm not, the finite element solution does not well approximate the exact solution in the vicinity of the boundary-and interior layers and even in the entire domain. Here “large” is of course a relative and user-tunable concept, depending on one or more physical parameters, such as the diffusivity, the convection field, the reaction, and even the mesh size. The solution of the convection-dominated problem is indeed in $H^2$ space, but due to the large $H^1$ semi-norm, I view it as quasi non $H^1$ space very weak solution, although the really non $H^1$ space very weak solution usually lives with infinity $H^1$ semi-norm. I will report our new multiscale and stabilization finite element method for convection-dominated problems. Numerical tests for a series of benchmark problems yield excellent numerical results and the boundary-and interior layers are clearly visible on uniform meshes with quite coarser mesh sizes. I emphasize that the solution in the benchmark problem is really non $H^1$ space very weak solution due to discontinuous boundary data, although the theoretical analysis still relies on the assumption that the $H^1$ and $H^2$ semi-norms of the solution are not large. The open problem is how to develop a finite element method, together with its mathematical theory, which can be really suitable for not only really non $H^1$ space very weak solution but also the quasi non $H^1$ space very weak solution like the solution that belongs $H^2$ space and even $H^2$ space but lives with boundary-and interior layers. Recently, I have developed $L^2$ projected finite element methods for really non $H^1$ space very weak solution with infinity $H^1$ semi-norm, with applications to Maxwell’s equations. As Part II of my work, I will report this for Maxwell’s equations in a later date in National University of Singapore.

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