The physics of information: from Maxwell’s demon to Landauer

Eric Lutz
University of Erlangen-Nürnberg
Outline

1. Information and physics
   - Information gain: Maxwell and Szilard
   - Information erasure: Landauer’s principle

2. Recent experimental realizations
   - Maxwell’s demon and Szilard engine
   - Landauer’s erasure principle

3. Thermodynamics of small systems
   - Jarzynski nonequilibrium work relation
   - Information erasure in small systems
Maxwell’s demon

Gas in a partitioned box

Maxwell 1867/1871

Information about particle (position/velocity) leads to sorting

→ temperature difference (decrease of entropy without work)
   (could be used to run a heat engine and produce work)

→ apparent violation of the second law
Szilard engine

One-particle gas in a partitioned box

Szilard 1929

Clear illustration of information–work relation

Maximum work: $W = kT \ln 2$ (reversible process)
What is information?

→ what you don’t already know!

Examples:

- you don’t learn much from "today is Friday"
  but you learn a lot from "oil price will double tomorrow"

- particle in a partitioned box

Information about the system is gained by looking into the box
"Information is physical "

- Information is transmitted by physical means
  - e.g. electromagnetic or mechanical signals

- Information is stored in physical systems
  - e.g. in a two-state system: left/right, up/down, hole/no hole, magnetization/no magnetization
  - bit = smallest unit of information

→ Information has to obey the laws of physics
Landauer erasure principle

"There is a price in forgetting"  
Landauer 1961

Erasure of information requires minimum heat dissipation

\[ \langle Q \rangle \geq \langle Q \rangle_{\text{Landauer}} = kT \ln 2 \]  
(per bit)

Important because:

- fundamental link between information and thermodynamics
- resolves Maxwell’s demon paradox  
  Bennett 1982

entropy (heat) dissipated larger than entropy decrease (work gained)

→ no violation of the 2nd law if memory reset is considered

- technological implications
Elementary derivation

Memory erasure = reset to state zero or one (reinitialization)

Ideal gas: \( P = \frac{NkT}{V} \)

Work during (quasistatic) compression:

\[
\langle W \rangle = -\int_{V}^{V/2} P \, dV = -NkT [\ln(\frac{V}{2}) - \ln V] = NkT \ln 2
\]

Cyclic process: \( \Delta U = -\langle Q \rangle + \langle W \rangle = 0 \)

\[
\langle Q \rangle \geq \langle Q \rangle_{\text{Landauer}} = kT \ln 2 \quad (N = 1)
\]
Demon performs single particle experiments

→ only a gedanken experiment?

"We never experiment with just one electron or atom or (small) molecule. In thought experiments we sometimes assume that we do; this invariably entails ridiculous consequences... In the first place it is fair to say that we cannot experiment with single particles, any more than we can raise ichtkyosauria in the zoo."

Schrödinger 1952

Well, not quite ....
Realization of Maxwell’s demon

Neutral atoms ($^{87}\text{Rb}$) in a dipole trap

Potential: $\sim \frac{I}{\Delta}$ with $\Delta < 0$ ($> 0$) for $F = 1$ ($2$) $\rightarrow$ well (barrier)

[Diagram showing the process of particle sorting]

$\rightarrow$ Demonstration of particle sorting ($\sim$ atomic diode)
Realization of the Szilard engine

Brownian particle in a tilted periodic potential

Toyabe et al. Nature Phys. 2010

Demonstration of information-to-work conversion
Verification of Landauer’s principle

Brownian particle in a double-well potential

Ciliberto ENS Lyon

Measured erasure cycle:

Landauer’s original thought experiment
Verification of Landauer’s principle

Generic two-state memory:

- initial configuration: two states with equal probability $1/2$
  - system can store 1 bit of information
  
  Shannon entropy: $S_i = - \sum p_n \ln p_n = \ln 2$

- final configuration: one state with probability 1
  - system can store 0 bit of information
  
  Shannon entropy: $S_f = - \sum p_n \ln p_n = 0$

  → original bit has been deleted: $\Delta S = - \ln 2$
More general derivation

Second law of thermodynamics (for system and reservoir):

\[ \Delta S = \Delta S_{sys} + \Delta S_{res} \geq 0 \]

Reservoir always in equilibrium:

\[ Q_{res} = T\Delta S_{res} \geq -T\Delta S_{sys} \]

Equivalence between entropies:

\[ \Delta S_{sys} = k\Delta S = -k \ln 2 \]

Heat produced in reservoir:

\[ Q_{res} \geq kT \ln 2 \]

connection between information theory and thermodynamics

\( Q_{res} = kT \ln 2 \) in quasistatic limit i.e. long cycle duration
Verification of Landauer’s principle

Experimental results:

We measure work $W$ and deduce heat $Q = -\Delta U + W = W$

→ Landauer can be bound approached but not exceeded

Note: $kT \ln 2 \approx 3 \times 10^{-21} J$ at room temperature
Logical irreversibility

Logical gates with more input states than output state

→ are logically irreversible

Examples:

- RESET TO ONE operation (information erasure)
- but also AND, NAND, OR, XOR, ....

General formulation of Landauer’s principle:

Any irreversible logical transformation of classical information produces at least $kT \ln 2$ of heat per bit
Technological consequences

Energy dissipation per logic operation:

Landauer Nature 1988


Pop Nano Research 2010

(Switching energy of silicon transistors)

Heating main issue hindering miniaturization
1. Information and physics
   - Information gain: Maxwell and Szilard
   - Information erasure: Landauer’s principle

2. Recent experimental realizations
   - Maxwell’s demon and Szilard engine
   - Landauer’s erasure principle

3. Thermodynamics of small systems
   - Jarzynski nonequilibrium work relation
   - Information erasure in small systems
Equilibrium (nonequilibrium) processes:

Entropy: \( \Delta S = \langle > \rangle \frac{Q}{T} \)

Work: \( W = \langle > \rangle \Delta F \quad (F = U - TS = \text{free energy}) \)

\( \rightarrow \) second law of thermodynamics

Properties:

- valid for all large systems \( \rightarrow \) universal theory
- for large systems, fluctuations are negligible \( \sim 1/\sqrt{N} \)

\( \rightarrow \) \( W \) and \( Q \) are deterministic variables
Thermodynamics of small systems

For small systems, fluctuations are important

$W$ and $Q$ are stochastic variables

Microsphere in a static optical trap:  
Imparato et al. PRE (2007)

→ second law has to be generalized
(replace inequality like $\langle W \rangle \geq \Delta F$ by equality)
Jarzynski equality

\[ \langle \exp(-\beta W) \rangle = \exp(-\beta \Delta F) \]

\[ \rightarrow \text{equilibrium free energy from nonequilibrium work } P(W) \]
\[ \rightarrow \text{valid arbitrarily far from equilibrium} \]
\[ \text{(beyond linear response)} \]

Properties:

- on average: \[ \langle W \rangle \geq \Delta F \]
  \[ \rightarrow \text{generalization of the second law} \]

- however: \[ P[W < \Delta F - \varepsilon] \leq \exp(-\varepsilon/(kT)) \]
  \[ \rightarrow \text{microscopic "violations" are exponentially small} \]
Experiments on Jarzynski

Stretching of single RNA molecule


![Diagram of RNA stretching](image)

**Work:**

\[
W = \int_{0}^{t} dt' \dot{\lambda} \partial_{\lambda} V(x(t'), \lambda)
\]

along a trajectory \(x(t)\)

Mechanical torsion pendulum

Douarche et al. EPL (2006)

![Diagram of torsion pendulum](image)

Colloid in nonharmonic potential

Blickle et al. PRL (2006)

![Diagram of colloid](image)
Applying Jarzynski to Landauer’s problem:

- on average: \( \langle Q \rangle = \langle W \rangle \geq \Delta F = kT \ln 2 \)
- however: \( P[Q < Q_{\text{Landauer}} - \varepsilon] \leq \exp(-\varepsilon/(kT)) \)

\[ \Rightarrow \] full information erasure below Landauer is possible (for single realizations) [Dillenschneider, Lutz PRL (2009)]

Measured heat distribution:
Connection between information and thermodynamics

- Maxwell, Szilard and Landauer

Have all been realized experimentally

- from gedanken to real experiments

Thermal fluctuations are important at the micro/nanoscale

- second law has to be generalized (Jarzynski equality, fluctuation theorem,...)
Quantum heat engines


- Efficiency of heat engines coupled to nonequilibrium reservoirs  arXiv:1303.6558

- A nano heat engine beyond the Carnot limit  arXiv:1308.5935