The computable Lipschitz reducibility and the uniformly non-low_2 c.e. degrees

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Measures of relative randomness

- In randomness and incomputability we have two fundamental measures: the plain complexity C and the prefix-free complexity K.

- Real α is Δ_2^0 (c.e.) if it is the limit of a computable (increasing) sequence of rational numbers.

- $\alpha \leq_K \beta$ if $K(\alpha \upharpoonright n) \leq K(\beta \upharpoonright n) + O(1)$.

- $\alpha \leq_C \beta$ if $C(\alpha \upharpoonright n) \leq C(\beta \upharpoonright n) + O(1)$.

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Definition (Downey,Hirschfeldt,2008)

Given two reals α and β, α is computable Lipschitz ($\leq_{cl}$) to β if there is a Turing functional Γ and a constant c such that $\alpha = \Gamma^{\beta}$ and the use of Γ on any argument n is bounded by $n + c$.

Proposition (Downey,Hirschfeldt and Lafort,2008)

If $\alpha \leq_{cl} \beta$, then for all n,

$$K(\alpha \upharpoonright n) \leq K(\beta \upharpoonright n) + O(1)$$

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The cl-degree only contains either only random reals or non-random reals.

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The cl-degrees of c.e. reals is neither a lower semi-lattice, nor an upper semi-lattice.

Theorem (Yu and Ding,2004)

There is no cl-complete c.e. real.

Corollary (Yu and Ding,2004)

There are two c.e.reals α and β which have no common upper bound under cl-reducibility in c.e. reals.

Theorem (Barmpalias and Levis,2006)

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For a c.e. degree **d**, the following are equivalent:

(1) **d** is array non-computable.

(2) There are c.e. reals $\alpha, \beta \in \mathbf{d}$ which have no common upper bound in the cl-degrees of c.e. reals.

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(A, B) is a cl-maximal pair of c.e. sets if no c.e. set can cl-compute both of them.

Proposition (Barmpalias,2005; Fan and Lu,2005)

There exists a cl-maximal pair of c.e. sets.

Theorem (Ambos-spies, Ding, Fan and Wolfgang, 2013)

For any c.e. set D the following are equivalent:

(1) $Deg_T(D)$ is array non-computable.

(2) There is a cl-maximal pair (A, B) such that $A \equiv_T B \equiv_T D$.

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We focus on the c.e. Turing degrees and continue this line of investigation.

A Turing degree $\mathbf{d}$ is array non-computable if for any total function $f \leq_{wtt} \emptyset'$ there is a total function $g \leq_T \mathbf{d}$ which is not dominated by f.

A Turing degree $\mathbf{d}$ is non-low_2 if for any total function $f \leq_T \emptyset'$ there is a total function $g \leq_T \mathbf{d}$ which is not dominated by f.

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Corollary (Fan and Yu,2012)

Each non-computable c.e. real is the half of a cl-maximal pair of c.e. reals.

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Uniformly non-low_2

A c.e. Turing degree $\mathbf{d}$ is uniformly non-low_2 if there is a computable function h so that if the function $\Phi_e^{\emptyset'}$ is total then $\Phi_{h(e)}^{\mathbf{d}}$ is total and not dominated by $\Phi_e^{\emptyset'}$. We say h is the uniform function for $\mathbf{d}$.

Proposition

There is an incomplete uniformly non-low_2 c.e. degree $\mathbf{d}$.

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$$\{\text{uniformly non}-\text{low}_2\} \subsetneq \{\text{non}-\text{low}_2\} \subsetneq \{\text{array non}-\text{computable}\}.$$

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If a c.e. Turing degree $\mathbf{d}$ is uniformly non-low_2, for any non-computable Δ_2^0 real α there is a c.e. real $\beta \in \mathbf{d}$ so that no c.e. real can cl-compute both of them.

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- $R_e : \alpha \neq \Gamma_{e_1}^{\gamma_{e_0}}$ *or* $\beta \neq \Gamma_{e_2}^{\gamma_{e_0}}$ for $e = \langle e_0, e_1, e_2 \rangle$.
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Corollary 1

If a c.e. Turing degree $\mathbf{d}$ is uniformly non-low_2,
for each c.e. real $\alpha \in \mathbf{d}$ there is a c.e. real $\beta \in \mathbf{d}$ so that (α, β) is a cl-maximal pair of c.e. reals.

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Definition (Kjos-Hanssen, Wolfgang, Stephen, 2006)

A set A is complex if there is an order (nondecreasing, unbounded, computable) function h such that $K(A \upharpoonright x) > h(x)$ for all x.

Proposition (Downey, Hirschfeldt,2004)

There is a real (not c.e.) which is not cl-reducible to any random real (indeed to any complex real).

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A degree $\mathbf{d}$ is called generalised low_2 if $\mathbf{d}'' \leq (\mathbf{d} \vee \mathbf{0}')'$.

Proposition (Barmpalias, Downey, Greenberg, 2010)

For any non-generalised-low_2 degree $\mathbf{d}$, there is some $A \leq_T \mathbf{d}$ which is not cl-reducible to any complex real.

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Theorem 3

Each uniformly non-low_2 c.e. Turing degree contains a c.e. real which is not cl-reducible to any complex c.e. real.

Corollary 3

Each uniformly non-low_2 c.e. Turing degree contains a c.e. real which is not cl-reducible to any wtt-complete c.e. real.

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Open questions

- Which c.e. Turing degrees contain c.e. reals which are not *cl*-reducible to complex c.e. reals?
- Is there any characterization of the uniformly non-low_2 c.e. Turing degrees by *cl*-reducibility?
- More properties of the uniformly non-low_2 c.e. degrees.

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Thank you!