FROM RANDOMNESS TO ORDER

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Abstract

A major achievement of the Nobel prize winning economist Thomas Schelling was an elegant model of racial segregation, first described in 1969. In this model, one starts with a random arrangement of two types of objects (in one or two dimensions) and initiates a stochastic process which rearranges the objects according to some rules, which induce a probabilistic (nondeterministic) process. The aim of the model was to understand the large scale segregation observed between black and white communities in American cities. However the model can be used to interpret other phenomena such as inter-particle forces.

Since the 1970s there have been a number of studies of this model. However, these are geared towards experiments (computer simulations) or use modified (perturbed) versions of the model which make the corresponding Markov process more regular and suitable for an analysis via methods of statistical mechanics and evolutionary game theory.

The first theoretical analysis of the actual (one dimensional) model appeared recently in a paper by Brandt, Immorlica, Kamath and Kleinberg, which dealt with a specific case. For the first time we have rigorously and globally analysed the one-dimensional Schelling model and have achieved a deeper understanding of its interesting behaviour, such as situations where an increase in racial tolerance increases segregation. Along with this theoretical analysis, we have produced a number of computer simulations which illustrate our theorems. A picture produced via such simulations recently won the Picturing Science competition of the Royal Society.

One of our graphical representations is demonstrated with two examples in Figure 1. Here two types of individuals (coloured black and grey) are arranged randomly in a circle in the centre of the large circle (which is collapsed to a point in the figure). During the stages of the process, individuals swap places with the incentive to move to a neighbourhood with higher concentration of individuals of their own colour. A key parameter is the proportion of nodes of a different colour which any node will tolerate within their neighbourhood. Once this threshold is passed, they become unhappy and desire to move out. This is the intolerance is a parameter of the system, which takes values from 0 to 1 and affects the behaviour of the system significantly. The main goal of this research is to classify the behaviour of the system (and in particular the final state) according to the value of the intolerance. In our graphical representation, each swap is represented with a dot between the (large) circle and its centre, with colour the latest colour of that position which changed inhabitant. Moreover the dots are placed at a distance from the centre which is analogous to the time where the change happened. Hence dots that occur closer to the centre depict swaps that occurred earlier in the process. The outer (large) circle shows the state of the system at the end of the process.

This 2-dimensional representation allows a record of the entire series of states of the one-dimensional system. The two pictures correspond to the processes with different intolerance parameters, 0.485 and 0.49 respectively. In both cases, cascades of swaps of the same type are clearly visible. On the other hand, such cascades are more

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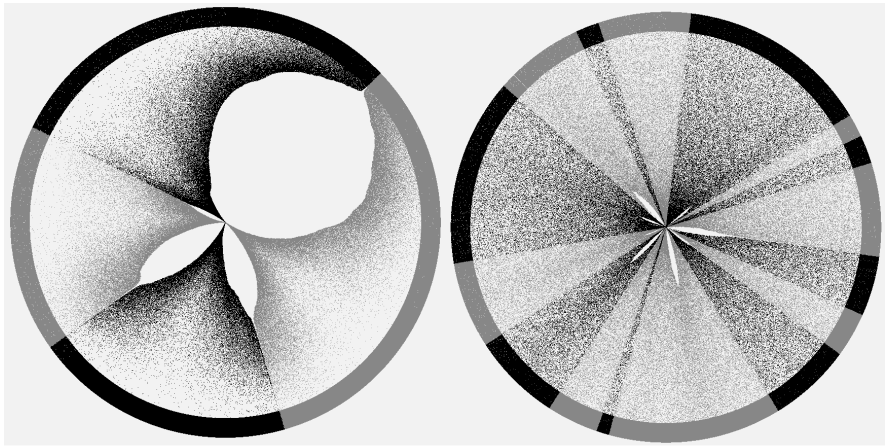

FIGURE 1. Schelling segregation in one dimension: population a million. The paradox of increased tolerance leading to increased segregation.

restricted in the second figure. This is a demonstration of a paradox that was discussed empirically by Schelling more than 40 years ago: in many cases increased tolerance leads to increased segregation. In our work we have rigorously proved this phenomenon and specified the conditions that are required for its occurrence. The outcomes of computer simulations correspond rather precisely to parts of the theory that we developed in order to prove rigorous results for the outcome of the process, depending on the parameters.

In this talk I will present our results and illustrate them via a number of computer simulations (pictures and animations). This is joint work with Richard Elwes (University of Leeds) and Andy Lewis-Pye (London School of Economics).

Our papers can be downloaded from

`http://barmpalias.net/papers/Schel1D.pdf`

`http://arxiv.org/pdf/1311.5934`

`http://barmpalias.net/papers/minority.pdf`

Some code for fast simulations can be found at:

`http://barmpalias.net/schelcode.shtml`

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