Nondeterministic automatic complexity of almost square-free and strongly cube-free words

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Abstract. Shallit and Wang studied deterministic automatic complexity of words. They showed that the automatic Hausdorff dimension $I(\mathbf{t})$ of the infinite Thue word satisfies $1/3 \le I(\mathbf{t}) \le 2/3$. We improve that result by showing that $I(\mathbf{t}) \ge 1/2$. For nondeterministic automatic complexity we show $I(\mathbf{t}) = 1/2$. We prove that such complexity A_N of a word x of length n satisfies $A_N(x) \le b(n) := \lfloor n/2 \rfloor + 1$. This enables us to define the complexity deficiency $D(x) = b(n) - A_N(x)$. If x is square-free then $D(x) = 0$. If x almost square-free in the sense of Fraenkel and Simpson, or if x is a strongly cube-free binary word such as the infinite Thue word, then $D(x) \le 1$. On the other hand, there is no constant upper bound on D for strongly cube-free words in a ternary alphabet, nor for cube-free words in a binary alphabet.
The decision problem whether $D(x) \ge d$ for given x, d belongs to NP$\cap$E.