Schnorr-randomness versions of $K$, $C$, LR, vL-reducibilities

Kenshi Miyabe
Meiji University, Japan
research@kenshi.miyabe.name

One of natural measures of randomness is $K$-reducibility, which is defined by $X \leq_K Y$ if and only if

$$K(X \upharpoonright n) \leq K(Y \upharpoonright n) + O(1).$$

Much effort has been devoted to the study of this and related reducibilities.

One direction to analyse $K$-reducibility is the one via lowness. In fact, $X \leq_K \emptyset$ if and only if $X \leq_{LR} \emptyset$ if and only if $X \leq_{LK} \emptyset$, which is shown by Nies [8]. Kjos-Hanssen et al. [2] strengthened this result to that $LK$-reducibility is actually equivalent to $LR$-reducibility.

Another important reducibility to analyse $K$-reducibility is $vL$-reducibility. For ML-random sets, $vL$-reducibility is the converse of $LR$-reducibility. Miller and Yu [4] showed that $X \leq_K Y$ implies $X \leq_{vL} Y$.

We consider Schnorr-randomness versions of these results. Schnorr reducibility is the Schnorr-randomness version of $K$-reducibility, which is defined by $X \leq_{Sch} Y$ if for every computable measure machine $M$ there is a computable measure machine $N$ such that $K_N(X \upharpoonright n) \leq K_M(Y \upharpoonright n) + O(1)$.

Kjos-Hanssen et al. [3] showed that a set is low for Schnorr randomness if and only if it is computably traceable. Then, Nies [9, Problem 8.4.22] asked whether the reducibility versions are equivalent or not. We answer this affirmatively using the open covering method developed by Bienvenu and Miller [1]. A similar result was obtained in [5] with uniform relativization.

Miyabe [6] and Miyabe and Rute [7] showed van Lambalgen’s theorem for uniform Schnorr randomness. Thus, we can consider a Schnorr-randomness version of $vL$-reducibility. We show that Schnorr reducibility implies the Schnorr-randomness version of $vL$-reducibility. The key of the proof is an extension of the Ample Excess Lemma.

References