ON ROGERS SEMILATTICES OF ANALYTICAL HIERARCHY

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We investigate some algebraic properties of Rogers semilattices of analytical hierarchy: existence of minimal elements, ideals without minimal elements. For an at most countable non-empty family $S$ of subjects of the natural series, its numbering $\alpha : \mathbb{N} \rightarrow S$ is said to be $\Sigma_{n+1}^1$-computable if the set $\{(x, y) \mid x \in \alpha(y)\} \in \Sigma_{n+1}^1$. The set of all $\Sigma_{n+1}^1$-computable numberings of the family $S$ is denoted by $Com_{n+1}^1(S)$. Enumeration $\nu \in Com_{n+1}^1(S)$ is called minimal, if for every $\mu \in Com_{n+1}^1(S)$ such that $\mu \leq \nu$, performed $\nu \equiv \mu$. One of the most important minimal numberings is Friedberg's numbering. Owings showed in [2] that there is no $\Pi_1^1$-computable Friedberg enumeration of all $\Pi_1^1$-sets using metarecursion theory. This result is obtained in classic computability theory for higher levels of analytical hierarchy.

**Theorem**

1. There are infinitely many minimal numberings of an infinite family $S$ of $\Pi_{n+1}^1$-sets.
2. There is no a $\Pi_{n+1}^1$-computable Friedberg enumeration of all $\Pi_{n+1}^1$-sets.
3. Elementary theory of any nontrivial Rogers semilattices of analytical hierarchy is undecidable.
4. Let $S$ be infinite family of $\Sigma_{n+1}^1$-sets, $Com_{n+1}^1(S) \neq \emptyset$. Then there exists a numbering $\beta \in Com_{n+1}^1(S)$ such that $\hat{\beta}$ (the principal ideal of Rogers semilattices $\mathcal{R}_{n+1}^1(S)$ generated by $\deg(\beta)$) contains no minimal elements.

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**References**


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