Cone avoidance and randomness preservation

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This is joint work with Frank Stephan. Our paper “Cone avoidance and randomness preservation” has been conditionally accepted for publication in Annals of Pure and Applied Logic.

Let $X$ be an infinite sequence of 0’s and 1’s. Let $f$ be a computable function. Recall that $X$ is strongly $f$-random if and only if the a priori Kolmogorov complexity of each finite initial segment $\tau$ of $X$ is bounded below by $f(\tau)$ minus a constant. We study the problem of finding a PA-complete Turing oracle which preserves the strong $f$-randomness of $X$ while avoiding a Turing cone. In the context of this problem, we prove that the cones which cannot always be avoided are precisely the K-trivial ones. We also prove: (1) If $f$ is convex and $X$ is strongly $f$-random and $Y$ is Martin-Löf random relative to $X$, then $X$ is strongly $f$-random relative to $Y$. (2) $X$ is complex relative to some oracle if and only if $X$ is $\mu$-random for some continuous probability measure $\mu$. 