Cone avoidance and randomness preservation

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This is joint work with Frank Stephan. Our paper "Cone avoidance and randomness preservation" has been conditionally accepted for publication in Annals of Pure and Applied Logic.

Let X be an infinite sequence of 0's and 1's. Let f be a computable function. Recall that X is strongly f-random if and only if the a priori Kolmogorov complexity of each finite initial segment τ of X is bounded below by $f(\tau)$ minus a constant. We study the problem of finding a PA-complete Turing oracle which preserves the strong f-randomness of X while avoiding a Turing cone. In the context of this problem, we prove that the cones which cannot always be avoided are precisely the K-trivial ones. We also prove: (1) If f is convex and X is strongly f-random and Y is Martin-Löf random relative to X, then X is strongly f-random relative to Y. (2) X is complex relative to some oracle if and only if X is μ-random for some continuous probability measure μ.