

Triviality within and beyond hyperarithmetic

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It is known that every non-computably-dominated real must compute a weakly 1-generic real. However, it is known in set theory that several forcing notions such as the rational perfect forcing and the Laver forcing add an unbounded real without adding a Cohen real. By simulating the rational perfect forcing construction over $L_{\omega_1^{\text{CK}}}$, the ω_1^{CK} -th rank of Gödel's constructible universe, we show that there is a non- Δ_1^1 -dominated real which hyp-computes only Δ_1^1 -trivial reals (hence, hyp-computes no Δ_1^1 -generic real). By combining this argument with local Cohen forcing, we also show that the same property holds in the E_n -degrees, where $E_0 =^2 E$ is Kleene's normal type 2 functional, and E_{n+1} is the superjump of E_n . Moreover, we show that the same property holds in the Δ_n^1 -degrees under the projective determinacy.