Online Boosting Algorithms

Satyen Kale

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- At the end, predict by taking a (weighted) majority vote.

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An natural question: how to extend boosting to the online setting?

Related Work

Several algorithms exist (Oza and Russell, 2001; Grabner and Bischof, 2006; Liu and Yu, 2007; Grabner et al., 2008).

- mimic offline counterparts.
- achieve great success in many real-world applications.
- no theoretical guarantees.

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Chen et al. (2012): first online boosting algorithms with theoretical guarantees.

- online analogue of weak learning assumption.
- connecting online boosting and smooth batch boosting.

Online Boosting for Classification

Alina Beygelzimer[1] Satyen Kale[1] Haipeng Luo[2]

[1]Yahoo! Labs, NYC

[2]Computer Science Department, Princeton University

Batch Boosting

Given a *batch* of T examples, $(\mathbf{x}_t, y_t) \in \mathcal{X} \times \{-1, 1\}$ for $t = 1, \ldots, T$.
Learner predicts $\hat{y}_t \in \{-1, 1\}$ for example $\mathbf{x}_t$.

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$$\Big\Downarrow \text{(Schapire, 1990; Freund, 1995)}$$

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Learner observes $\mathbf{x}_t$ and predicts $\hat{y}_t \in \{-1, 1\}$ before seeing y_t.

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Weak Online learner (with edge γ and excess loss S):

$$\sum_{t=1}^{T} \mathbf{1}\{\hat{y}_t \neq y_t\} \leq (\tfrac{1}{2} - \gamma) T + S$$

Strong Online learner (with error rate ϵ and excess loss S')

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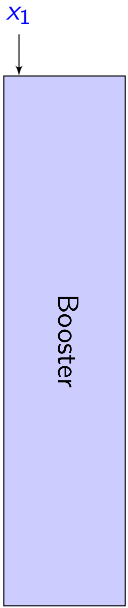
this talk: $S = \frac{1}{\gamma}$ (corresponds to $\sqrt{T}$ regret)

Main Results

Parameters of interest:
N = number of weak learners (of edge γ) needed to achieve error rate ϵ.
T_ϵ = minimal number of examples s.t. error rate is ϵ.

Algorithm	N	T_ϵ	Optimal?	Adaptive?
Online BBM	$O(\frac{1}{\gamma^2} \ln \frac{1}{\epsilon})$	$\tilde{O}(\frac{1}{\epsilon\gamma^2})$	$\surd$	$\times$
AdaBoost.OL	$O(\frac{1}{\epsilon\gamma^2})$	$\tilde{O}(\frac{1}{\epsilon^2\gamma^4})$	$\times$	$\surd$
Chen et al. (2012)	$O(\frac{1}{\epsilon\gamma^2})$	$\tilde{O}(\frac{1}{\epsilon\gamma^2})$	$\times$	$\times$

Structure of Online Boosting



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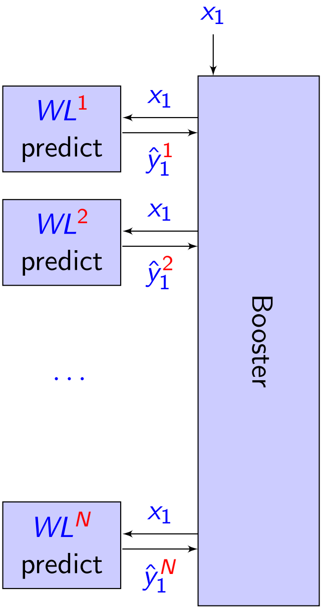

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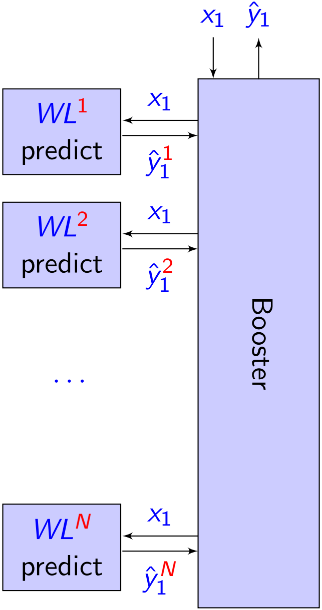

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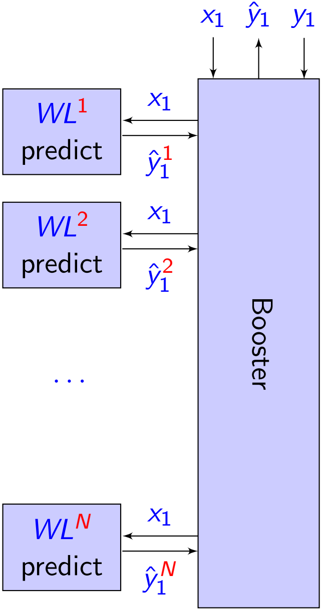

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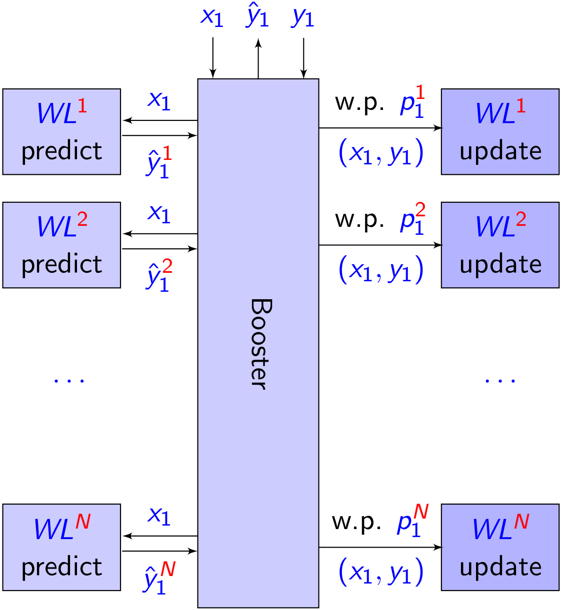

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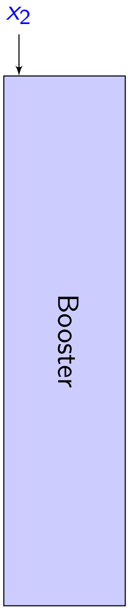

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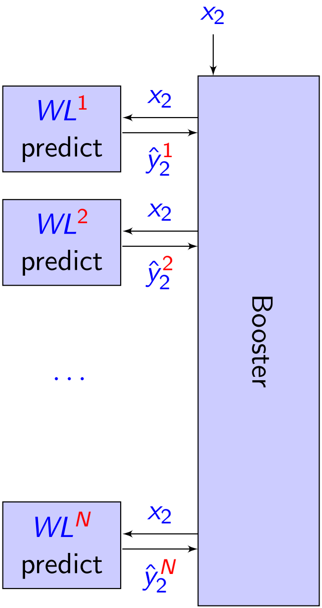

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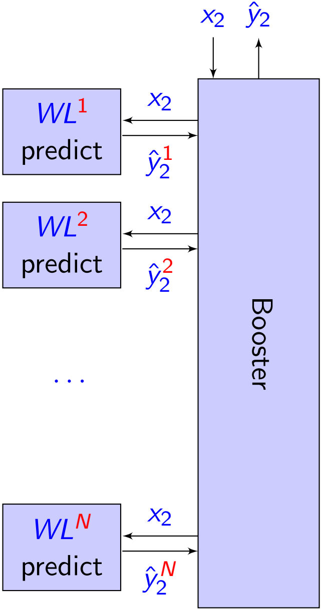

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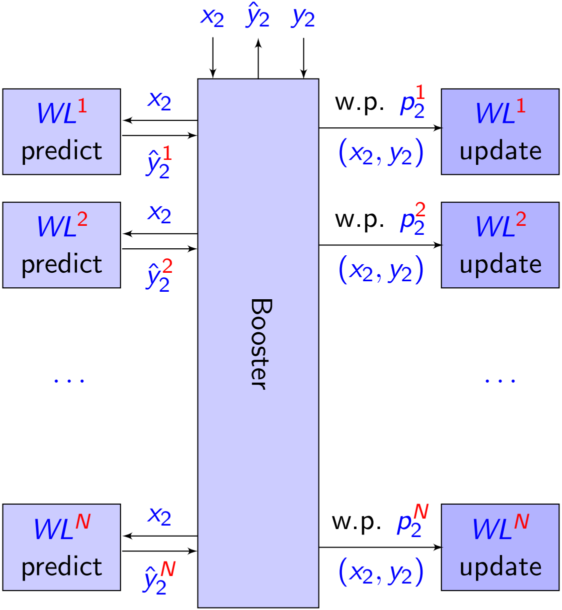

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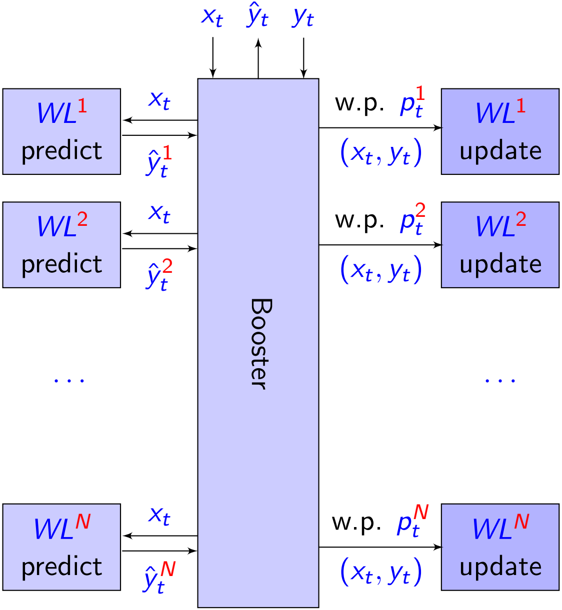

Boosting as a Drifting Game (Schapire, 2001; Luo and Schapire, 2014)

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Online version: sequence of potentials $\Phi_i(s)$ s.t.

$$\begin{aligned} \Phi_N(s) &\geq \mathbf{1}\{s \leq 0\}, \\ \Phi_{i-1}(s) &\geq (\tfrac{1}{2} - \tfrac{\gamma}{2})\Phi_i(s-1) + (\tfrac{1}{2} + \tfrac{\gamma}{2})\Phi_i(s+1). \end{aligned}$$

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Online boosting algorithm using Φ_i:

- **prediction**: majority vote.
- **update**: $p_t^i = \Pr[(\mathbf{x}_t, y_t) \text{ sent to } WL^i] \propto w_t^i$ where $w_t^i =$ difference in potentials if example is misclassified or not.

Mistake Bound

Generalized drifting games analysis implies

$$\sum_{t=1}^{T} \mathbf{1}\{\hat{y}_t \neq y_t\} \leq \underbrace{\Phi_0(0)}_{\leq \epsilon} T + \underbrace{(S + \tfrac{1}{\gamma}) \textstyle\sum_i \|\mathbf{w}^i\|_\infty}_{=S'}.$$

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Online BBM: to get ϵ error rate, needs
$N = O(\frac{1}{\gamma^2} \ln(\frac{1}{\epsilon}))$ weak learners and $T_\epsilon = O(\frac{1}{\epsilon\gamma^2})$ examples. (Optimal)

Drawback of Online BBM

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- treats each weak learner equally: predicts via simple majority vote.

Adaptivity is the key advantage of AdaBoost!

- different weak learners weighted differently based on their performance.

Adaptivity via Online Loss Minimization

Batch boosting finds a combination of weak learners to minimize some loss function using coordinate descent. (Breiman, 1999)

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We generalize it to the online setting:

- replace line search with online gradient descent.
- exponential loss does not work again, use logistic loss to get adaptive online boosting algorithm AdaBoost.OL.

Mistake Bound

If WL^i has edge γ_i, then

$$\sum_{t=1}^{T} \mathbf{1}\{\hat{y}_t \neq y_t\} \leq \frac{2T}{\sum_i \gamma_i^2} + \tilde{O}\left(\frac{N^2}{\sum_i \gamma_i^2}\right)$$

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Suppose $\gamma_i \geq \gamma$, then to get ϵ error rate AdaBoost.OL needs $N = O(\frac{1}{\epsilon\gamma^2})$ weak learners and $T_\epsilon = O(\frac{1}{\epsilon^2\gamma^4})$ examples.

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Not optimal but adaptive.

Results

Available in Vowpal Wabbit 8.0.

- command line option: `--boosting`.
- VW as the default "weak" learner (a rather strong one!)

Dataset	VW baseline	Online BBM	AdaBoost.OL	Chen et al. 12
20news	0.0812	0.0775	0.0777	0.0791
a9a	0.1509	0.1495	0.1497	0.1509
activity	0.0133	0.0114	0.0128	0.0130
adult	0.1543	0.1526	0.1536	0.1539
bio	0.0035	0.0031	0.0032	0.0033
census	0.0471	0.0469	0.0469	0.0469
covtype	0.2563	0.2347	0.2495	0.2470
letter	0.2295	0.1923	0.2078	0.2148
maptaskcoref	0.1091	0.1077	0.1083	0.1093
nomao	0.0641	0.0627	0.0635	0.0627
poker	0.4555	0.4312	0.4555	0.4555
rcv1	0.0487	0.0485	0.0484	0.0488
vehv2binary	0.0292	0.0286	0.0291	0.0284

Online Boosting for Regression

Alina Beygelzimer[1] Elad Hazan[2] Satyen Kale[1] Haipeng Luo[2]

[1]Yahoo! Labs, NYC

[2]Computer Science Department, Princeton University

Regression Setting

Setup:

- Examples $(\mathbf{x}, y) \in \mathcal{X} \times [-1, 1]$
- Loss of predicting $\hat{y}$ for $(\mathbf{x}, y)$ is $(y - \hat{y})^2$
- $\mathcal{F}$ is a base class of regressors $f : \mathcal{X} \to [-1, 1]$
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Typically, by "greedily fitting the residual," as in Basis Pursuit.

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Boosting $\equiv$ greedy stagewise algorithm for fitting of additive models.

- **Input:** a batch S of examples, number of boosting steps N, and step size parameter η.

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(Friedman, 2001; Mason et al., 2000)
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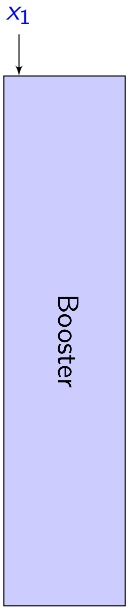
Structure of Online Boosting for Regression



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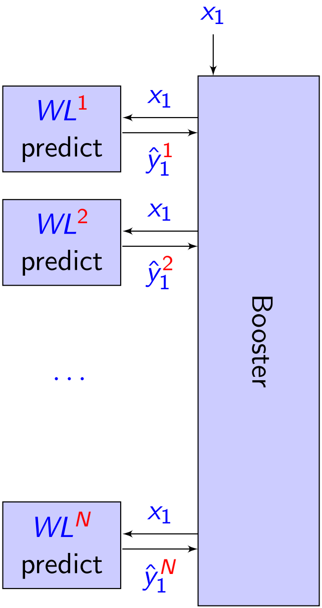

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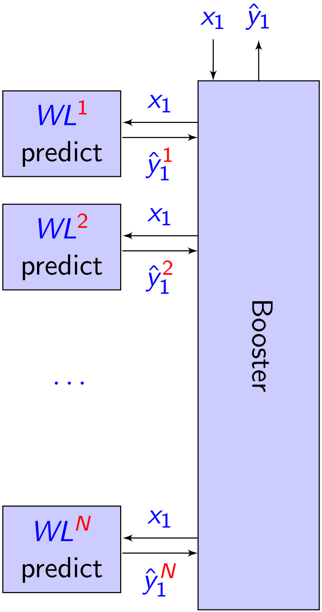

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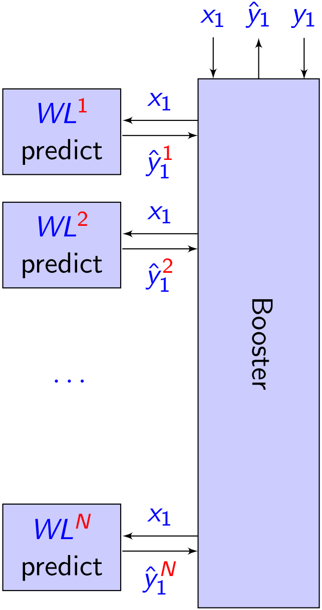

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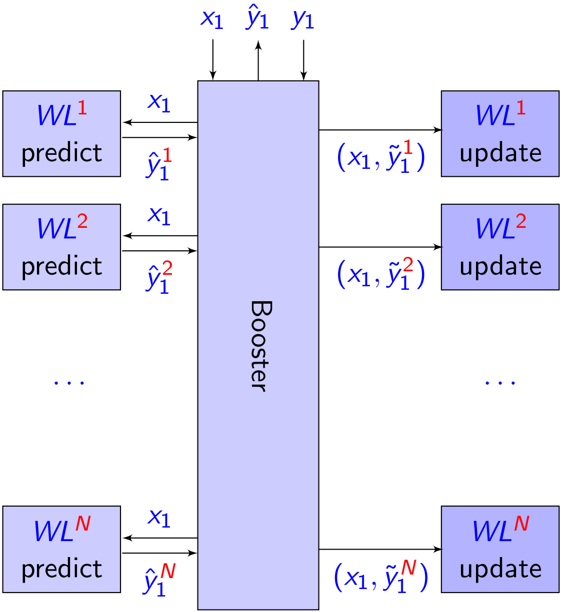

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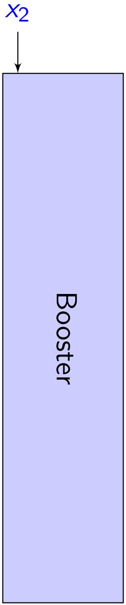

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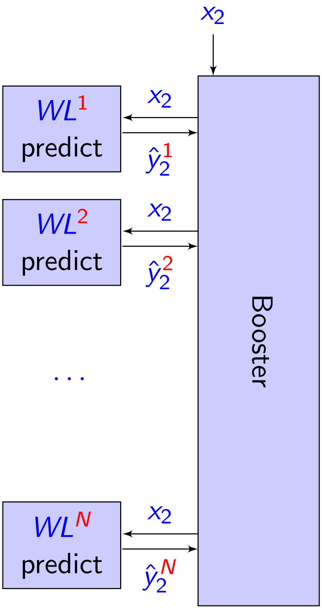

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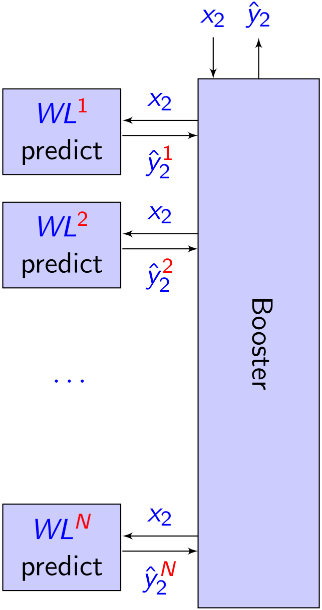

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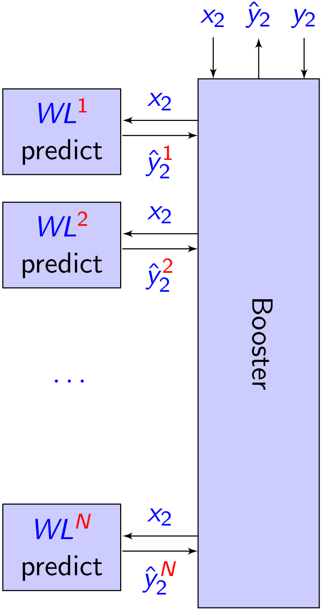

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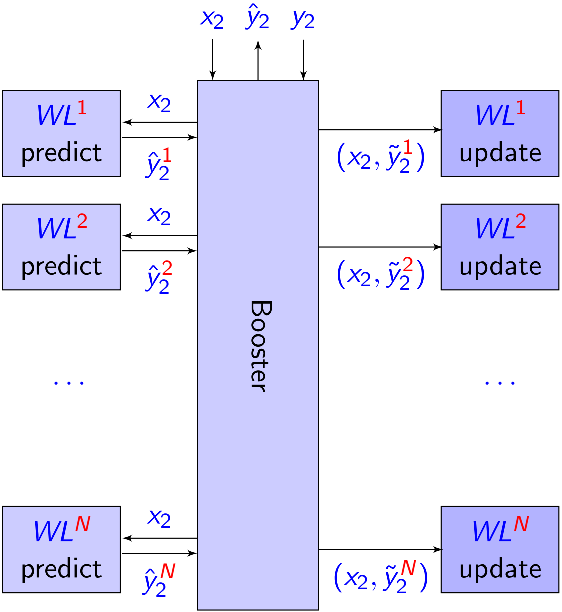

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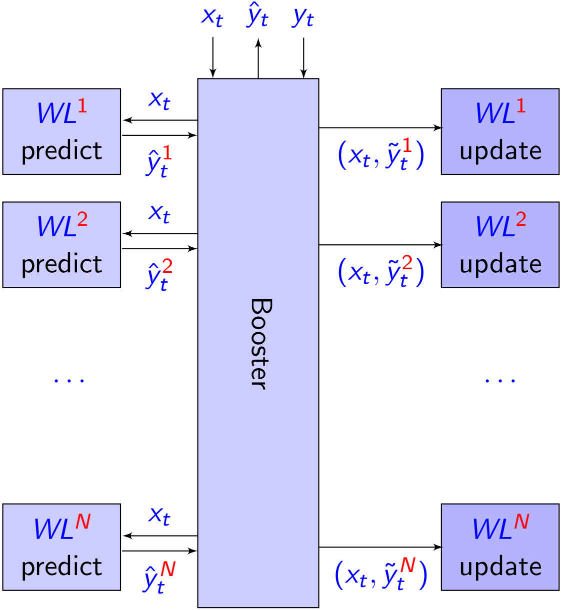

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$\sigma_t^i \in [0, \eta]$ are updated using gradient descent.

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For any $f \in \mathrm{span}(\mathcal{F})$,

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In **batch** setting, *exponentially faster convergence* compared to analysis of Zhang and Yu (2005).

Experiments

Setup:

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Base learner	**Average boost**	**Median boost**
VW	1.65%	0.03%
Neural networks	7.88%	0.72%
Regression stumps	20.22%	10.45%

Conclusions

We propose:

- A natural framework for online boosting.
- An optimal boosting algorithm for classification, Online BBM.
- An adaptive boosting algorithm for classification, AdaBoost.OL.
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Future directions:

- **Open problem:** optimal and adaptive boosting algorithm for classification?
- **Open problem:** is our regret bound in the regression setting tight?
- More experimentation and modifications for practical use.