Modeling and simulation of moving contact line problem for two-phase complex fluids flow

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4. Summary
When two immiscible fluids are in contact, the contact line forms as the intersection of the interface and the solid boundary.
Young’s Equation

The equilibrium configuration of the static contact line is described by Young’s equation (Young, 1805).

\[ \gamma \cos \theta_Y = \gamma_2 - \gamma_1, \]

**Figure:** Two immiscible fluids in contact. \( \gamma, \gamma_1 \) and \( \gamma_2 \) are the surface tension coefficients of the three interfaces; \( \theta \) is the contact angle of the fluid interface with the solid surface.
Wetting Phenomenon

Four types of wetting:

- $\theta_Y = 0^\circ$: complete wetting;
- $0 < \theta_Y < 90^\circ$: hydrophilic;
- $90 < \theta_Y < 180^\circ$: hydrophobic;
- $\theta_Y = 180^\circ$: no wetting.
It is well known that there is a non-integrable singularity at the moving contact line for corner flows with no slip boundary condition (Huh and Scriven, 1971):

$$F_{\text{shear}} = \int_{r=0}^{R} \mathbf{t} \cdot \tau_d(r) \cdot \mathbf{n} \, dr \sim \int_{r=0}^{R} \eta \frac{U}{r} \, dr = \infty$$

where $\tau_d(r)$ is the shear viscous stress measured at a distance $r$ to the contact line, $\mathbf{t}$ and $\mathbf{n}$ are the tangential and normal vector to the solid boundary, respectively.

One solution is to introduce nonzero slip velocity $u_s$ on the solid boundary:

$$\frac{\partial u_s}{\partial n} = -\beta u_s, \quad \beta > 0$$
Navier slip model eliminates the singularity of viscous stress (from $O(\frac{1}{r})$ to $O(1)$), make the pressure singularity integrable (from $O(\frac{1}{r})$ to $O(\log r)$).

Several generalizations:

- Alter the equivalent slip length scale near the contact line: Ruckenstein and Dunn 1977, Huh and Mason 1977, Zhou and Sheng 1990.
- Contact line condition and slip boundary condition in sharp interface formulation: Ren and E (2007).
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Industrial applications: hydrodesulfurization of crude oil, cleaning using detergent, printing and painting, micro fluidics, etc.
Two phase flows with surfactant on the closed interface have been well modeled and simulated:

- **Sharp interface model for insoluble surfactant**: boundary integral method (Yon et al, 1999); volume-of-fluid method (James and Lowengrub, 2004); level-set method (Xu et al, 2006); and immersed boundary method (Lai et al, 2008).

- **Model for soluble surfactant**: front tracking method (Muradoglu and Tryggvason, 2008), phase-field model (Liu and Zhang, 2010).

Less work is done on the contact line problem with surfactant: immersed boundary method (Lai et al, 2010), level-set method (Ren and Xu, 2014).
Major Mechanics

- The surface tension of the fluid interface depends on the concentration of the surfactant on the interface.
- This influences the velocity field of the fluids.
- In return, the flow changes the distribution of the surfactant on the interface through convection.

Methodology: Navier-Stokes + Thermodynamics with free energy

Figure: Droplet on a solid substrate.
The total free energy of the static droplet system is

\[ E(z, c) = \int_{\Gamma_1} (\gamma_1 - \gamma_2) \, dx \, dy + \int_{\Gamma_1} e(c) \sqrt{1 + z_x^2 + z_y^2} \, dx \, dy \]

\[ \text{fluid-solid interfacial energy} \quad \text{fluid-fluid interfacial energy} \]

The constraints are the fixed volume of the droplet and the fixed total amount of surfactant:

\[ \int_{\Gamma_1} z(x, y) \, dx \, dy = Q, \]

\[ \int_{\Gamma_1} c(x, y) \sqrt{1 + z_x^2 + z_y^2} \, dx \, dy = M. \]

Moreover, \( z(x, y) = 0 \) holds at the contact line \( \Lambda \).
Calculus of Variation

\[ L = \int_{\Gamma_1} \mathcal{L}(z, z_x, z_y, c) \, dx \, dy, \]

where

\[ \mathcal{L} = (\gamma_1 - \gamma_2) + (e(c) - \mu c) \sqrt{1 + z_x^2 + z_y^2 + \lambda z}. \]

The Euler-Lagrange equations are given by

\[
- \nabla \cdot \left( \frac{\partial \mathcal{L}}{\partial z_x}, \frac{\partial \mathcal{L}}{\partial z_y} \right) + \frac{\partial \mathcal{L}}{\partial z} = 0,
\]

\[ \frac{\partial \mathcal{L}}{\partial c} = 0, \]

The boundary variation must be taken into account:

\[ \mathcal{L} - (\mathbf{n}_l \cdot \nabla z) \left( \mathbf{n}_l \cdot \left( \frac{\partial \mathcal{L}}{\partial z_x}, \frac{\partial \mathcal{L}}{\partial z_y} \right) \right) = 0, \quad \text{on } \Lambda. \]
The Euler-Lagrange equations and the boundary conditions are reduced to

\[ \gamma(c) \kappa = \lambda, \]  
\[ e'(c) = \mu, \]  
\[ \gamma(c) \cos \theta \gamma = \gamma_2 - \gamma_1, \]

(Young-Laplace equation)
(Constant chemical potential)
(Young’s equation)

where \( \gamma(c) = e(c) - e'(c)c \), the Legendre transform of \( e(c) \), is the surface tension of the fluid-fluid interface. \( \lambda \) and \( \mu \) are Lagrange multipliers.
Dynamic equation of the surfactant concentration was derived from the conservation law on the fluid interface using surface calculus (Stone, 1990, Wong et al, 1996):

$$\frac{Dc}{Dt} + (\nabla_s \cdot \mathbf{u}) c = -\nabla_s \cdot \mathbf{J}_c, \quad \text{on } \Gamma$$

It is an instance of the generalized Reynolds transport equation on surfaces (Cermelli et al, 2005).
The total free energy including kinetic energy is

\[ F = \sum_{i=1,2} \int_{\Omega_i} \frac{1}{2} \rho_i |\mathbf{u}|^2 \, d\mathbf{x} + (\gamma_1 - \gamma_2) |\Gamma_1| + \int_{\Gamma(t)} e(c) \, dA, \]

The iso-thermal and incompressible fluids follows Navier-Stokes equation in the bulk \( \Omega_i \) (\( i = 1, 2 \)):

\[ \rho_i (\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot \tau_d, \]

\[ \nabla \cdot \mathbf{u} = 0, \]

with Newtonian viscous stress tensor:

\[ \tau_d = \eta_i \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^\top \right). \]
Express the surface element as the differentiation of \((s^1, s^2)\), the parametrization of the initial interface, with its Jacobian:

\[
dA = \|\mathbf{t}_1 \times \mathbf{t}_2\| ds^1 ds^2
\]

Its time derivative:

\[
\frac{\partial}{\partial t} \|\mathbf{t}_1 \times \mathbf{t}_2\| = (\nabla_s \cdot \mathbf{u}) \|\mathbf{t}_1 \times \mathbf{t}_2\|
\]

The surface divergence theorem for any vector \(F(s^1, s^2)\) defined on the bounded surface \(\Gamma\):

\[
\int_{\partial \Gamma} \mathbf{F} \cdot \mathbf{n}_s dl = \int_{\Gamma} (\nabla_s \cdot \mathbf{F} - 2\kappa \mathbf{F} \cdot \mathbf{n}) dA,
\]
Energy Dissipations

Calculation using surface integration by parts yields:

\[
\frac{dF}{dt} = \dot{F}_b + \dot{F}_s + \dot{F}_i + \dot{F}_c + \dot{F}_l
\]

\[
\dot{F}_b = -\sum_{i=1,2} \int_{\Omega_i} \eta_i |\nabla u|^2 \, dx, \text{ (Bulk viscous dissipation)}
\]

\[
\dot{F}_s = \sum_{i=1,2} \int_{\Gamma_i} \mathcal{P} (\tau_d \cdot n_b) \cdot u_s \, dA, \text{ (Dissipation on solid surface)}
\]

\[
\dot{F}_i = \int_{\Gamma(t)} u \cdot \left\{ [\tau_d - p I] \cdot n + \gamma(c) \kappa n - \nabla_s \gamma(c) \right\} \, dA, \text{ (Interfacial)}
\]

\[
\dot{F}_c = \int_{\Gamma(t)} e''(c) \nabla_s c \cdot J_c \, dA, \text{ (Surfactant diffusion dissipation)}
\]

\[
\dot{F}_l = \int_{\Lambda} u_l \left\{ \gamma(c) \cos \theta_d + (\gamma_1 - \gamma_2) \right\} \, dl, \text{ (Dissipation on contact line)}
\]
Constitutive Relations

Assume $\dot{F}_i = 0 \Rightarrow$ Interface jump condition:

$$[\tau_d - pl] \cdot n = -\gamma(c)\kappa n + \nabla_s \gamma(c),$$

Marangoni force

Nonpositive dissipations $\dot{F_s}, \dot{F_c}, \dot{F_l} \leq 0 \Rightarrow$ Boundary conditions + Surfactant flux constraint:

$$\mathcal{P} (\tau_d \cdot n_b) = f_i(u_s), \quad \text{on } \Gamma_i, \text{(Navier slip BC)}$$

$$\gamma(c) \cos \theta_d + (\gamma_1 - \gamma_2) = f_{CL}(u_l), \text{(Contact angle condition)}$$

$$e''(c) \nabla_s c \cdot J_c \leq 0. \text{(Surfactant flux constraint)}$$
The energy density of surfactant given by

\[ e(c) = \gamma^0 + RTc_\infty \left( \frac{c}{c_\infty} \ln \frac{c}{c_\infty} + \left(1 - \frac{c}{c_\infty}\right) \ln \left(1 - \frac{c}{c_\infty}\right) \right), \]

yields Langmuir equation of state:

\[ \gamma(c) = \gamma^0 + RTc_\infty \ln \left(1 - \frac{c}{c_\infty}\right). \]

for \(0 < c < c_\infty\) (maximum packing).

\[ e''(c) > 0 \Rightarrow J_c = -D \nabla_s c \quad \text{(Fick’s law)} \]
\[
\begin{aligned}
\rho_i \left( \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) &= -\nabla p + \frac{1}{Re} \nabla \cdot \tau_d, & \text{in } \Omega_i, \\
\nabla \cdot \mathbf{u} &= 0, & \text{in } \Omega_i, \\
We \left[ \frac{1}{Re} \tau_d - pl \right] \cdot \mathbf{n} &= -\gamma(c) \kappa \mathbf{n} + \nabla_s \gamma(c), & \text{on } \Gamma(t), \\
-\beta_i \mathbf{u}_s &= \eta_i l_s \partial_n \mathbf{u}_s, & \text{on } \Gamma_i, \\
\mathbf{u} \cdot \mathbf{n}_b &= 0, & \text{on } \Gamma_i, \\
-\beta_{CL} \mathbf{u}_l &= \frac{1}{Ca} \left( \gamma(c) \cos \theta_d + (\gamma_1 - \gamma_2) \right), & \text{on } \Lambda, \\
\frac{Dc}{Dt} + (\nabla_s \cdot \mathbf{u}) c &= \frac{1}{Pe} \nabla^2_s c, & \text{on } \Gamma(t), \\
\nabla_s c \cdot \mathbf{n}_s &= 0, \\
\gamma(c) &= 1 + k \ln(1 - c), \\
\dot{x} &= \mathbf{u}(x, t). \quad \text{(kinematic condition)}
\end{aligned}
\]
Dimensional Parameters

- $Re = \frac{\rho_2 U L}{\eta_2}$, $Ca = \frac{\eta_2 U}{\gamma_3}$, $We = ReCa$, $l_s = \frac{\eta_2}{\beta_2 L}$

- $Pe_s = \frac{LU}{D_s}$, $k = \frac{RT c_\infty}{\gamma_3}$, $\tilde{\beta}_l = \frac{\beta_l}{\eta_2}$
Numerical Method

- Assume small Reynolds number ⇒ Stokes equation;
- Unstructured $P_2-P_1$ finite element method + Front tracking;
- Solve the $u^{n+1}$ using $c^n$ and $x^n$;
- Update $x^n$ (markers) according to $u^{n+1}$;
- Redistribute markers using equal-arclength parametrization;
- Interpolate $c^n$ to the new mesh from the old mesh;
- Solve $c^{n+1}$ on the new mesh by 1D finite element method.
Finite Element Formulation

Weak form:

\[
\int_{\Omega_1 \cup \Omega_2} \nabla u : \nabla v \, dx - \int_{\Omega_1 \cup \Omega_2} p \nabla \cdot v \, dx + \sum_{i=1}^{2} \int_{\Gamma_i} \frac{\beta_i}{l_s} (u \cdot t)(v \cdot t) \, dA
\]

\[
= \int_{\Gamma(t)} (-\gamma(c) \kappa n + \nabla_s \gamma(c)) \cdot v \, dA
\]

\[-\int_{\Omega_1 \cup \Omega_2} q \nabla \cdot u \, dx = 0\]

where \((v \cdot n)|_{\Gamma_i} = 0\).

Dirichlet boundary condition for \(u|_{\Lambda}\) at each \(t^n\) from unbalance Young’s force.
Convergence test

Parameters: $Ca = 0.1$, $Pe = 10$, $l_s = 0.1$, $k = 1$, $\eta_1 = \eta_2 = 1$, $\beta_1 = \beta_2 = 1$, $\beta_{CL} = 1$ (Ren et al, 2010)

Computational domain: $(x, y) \in [-1, 1] \times [0, 1]$

Initially: Static $\theta_Y = 60^\circ$; $c=0.4$ on the left half and $c = 0.2$ on the right half.

Numerical: $h = 0.01, 0.02, 0.04, 0.08$, $\Delta t = 5 \times 10^{-5}$, $N_s = 121, 61, 31, 17$ markers. Boundary: $y = 1$ is Dirichlet, $x = -1$ and $x = 1$ are periodic.
Parameters: $Ca = 0.1$, $Pe = 10$,  
$l_s = 0.1$, $k = 1$, $\eta_1 = \eta_2 = 1$,  
$\beta_1 = \beta_2 = 1$, $\beta_{CL} = 1$ (Ren et al, 2010)  
Initially: Static $\theta_Y = 60^\circ$; $c = 0.4$ on the left half and $c = 0.2$ on the right half.  
Numerical: $h = 0.02$, $\Delta t = 0.001$, $N_s = 61$ markers. Finally: New equilibrium contact angle is $\theta_Y = 41.9^\circ$.  

Figure: Viscous dissipation (solid), dissipation on the solid (dotted), contact line dissipation (dash-dot), interfacial dissipation (dashed).
Initially: semicircular droplet (dotted curve) with $\theta_d = 90^\circ$; $c=0.4$ (left) and $c = 0.2$ (right); Static contact angle $\theta_Y = 60^\circ$.
Finally: $\theta_Y = 60^\circ$ (clean, solid curve) and $\theta_y = 44.9^\circ$ (surfactant, dashed curve).

Droplet profile at $t = 0.3$: 

![Droplet profile at t = 0.3](image_url)
Relaxation Dynamics of Droplets (Hydrophobic)

Initially: semicircular droplet (dotted curve) with $\theta_d = 90^\circ$; $c=0.4$ (left) and $c = 0.2$ (right); Static contact angle $\theta_Y = 120^\circ$.

Finally: $\theta_Y = 120^\circ$ (clean, solid curve) and $\theta_y = 136.0^\circ$ (surfactant, dashed curve).
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4 Summary
Existing work

Two ways to affect contact line dynamics:

- Change the effective viscosity: Min et al (J. Colloid Interface Sci., 2011), dependent on the flow geometry


Numerical study:
Spaid and Homsy (J. Non-Newtonian Fluid Mech., 1994), spin coating
Yue and Feng (J. Non-Newton. Fluid Mech., 2012), phase-field, finite element, Oldroyd-B, studied viscous bending
Polymer stress tensor, FENE-P

Modify Navier-Stokes equation through stress tensor:

\[ \mathbf{T} = \tau_d + \tau_p \]  \hspace{1cm} (8)


\[ \tau_p = \frac{\eta_p}{\tau} \left( \frac{1}{1 - \text{tr} \mathbf{A}/E_d} \mathbf{A} - \frac{1}{1 - 3/E_d} \mathbf{I} \right) \]  \hspace{1cm} (9a)

\[ \frac{\partial \mathbf{A}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{A} - (\nabla \mathbf{u}) \mathbf{A} - \mathbf{A} (\nabla \mathbf{u})^\top = \frac{1}{\tau} \left( \frac{1}{1 - 3/E_d} \mathbf{I} - \frac{1}{1 - \text{tr} \mathbf{A}/E_d} \mathbf{A} \right) \]  \hspace{1cm} (9b)

\( E_d \): extensibility of the dumbbells
\( \tau \): relaxation time
\( \eta_p \): zero-shear viscosity
Unified system of equations

Immersed boundary formulation, $Wi = \frac{U_T}{L}$

$$\rho (\partial_t u + (u \cdot \nabla)u) = -\nabla p + \frac{1}{Re} (\eta \nabla^2 u + \nabla \cdot \tau_p) + \frac{1}{We} f,$$

$$\nabla \cdot u = 0,$$

$$\tau_p = \frac{\eta_p}{Wi} \left( \frac{1}{1 - \text{tr}A/E_d} A - \frac{1}{1 - 2/E_d} I \right),$$

$$\frac{\partial A}{\partial t} + u \cdot \nabla A - (\nabla u)A - A(\nabla u)^\top = \frac{1}{Wi} \left( \frac{1}{1 - 2/E_d} I - \frac{1}{1 - \text{tr}A/E_d} A \right)$$

$$f(x, t) = \int_D F(s, t) \delta(x - X(s, t)) ds$$

$$F(s, t) = \frac{\partial (\gamma_3 t(s, t))}{\partial s} = \gamma_3 \kappa(s, t) n(s, t) |\partial_s X(s, t)|$$

$$\frac{\partial X(s, t)}{\partial t} = U(s, t) = \int_\Omega u(x, t) \delta(x - X(s, t)) dx$$
Interpolation of velocity:

\[
U_k^n = \sum_x u^n \delta_h(x - X_k^n) \Delta x \Delta y
\]  

(11)

\[
X_{k+1}^{n+1} = X_k^n + \Delta t U_k^n
\]  

(12)

The contact line markers:

\[
-\beta_{CL} \frac{X_k^{n+1} - X_k^n}{\Delta t} = \frac{1}{Ca} (\gamma_3 \cos \theta_d^n + (\gamma_1 - \gamma_2)), \quad k = 0, M.
\]  

(13)

2. Equal-arclength Redistribution of the interface markers;


\[
A^{n+1} = A^n + \Delta t (-H^n(A^n) + G^n(A^n))
\]  

(14)
\[ H_{i,j}^n(q) = (u^n \cdot \nabla q)_{i,j} \]
\[ = \max(u^n, 0) q^{-}_{x;i,j} + \min(u^n, 0) q^{+}_{x;i,j} + \max(v^n, 0) q^{-}_{y;i,j} + \min(v^n, 0) q^{+}_{y;i,j} \]
\[ q^{-}_{x;i,j} = \frac{1}{2\Delta x} (D^+_{x} q_{i-1,j} + D^+_{x} q_{i,j}) - \frac{\omega_{x-}}{2\Delta x} (D^+_{x} q_{i-2,j} - 2D^+_{x} q_{i-1,j} + D^+_{x} q_{i,j}) \]
\[ q^{+}_{x;i,j} = \frac{1}{2\Delta x} (D^+_{x} q_{i-1,j} + D^+_{x} q_{i,j}) - \frac{\omega_{x+}}{2\Delta x} (D^+_{x} q_{i+1,j} - 2D^+_{x} q_{i,j} + D^+_{x} q_{i-1,j}) \]
\[ \omega_{x-} = \frac{1}{1 + 2r^2_{x-}}, \quad r_{x-} = \frac{\epsilon + (D^-_{x} D^+_{x} q_{i-1})^2}{\epsilon + (D^-_{x} D^+_{x} q_{i})^2} \]
\[ G^n(Q) = \nabla u^n Q + Q(\nabla u^n)^\top + \frac{1}{Wi} \left( \frac{1}{1 - 2/E_d} I - \frac{1}{1 - \tr Q/E_d} Q \right) \]
Spread the force:

\[ f^{n+1}(x) = \sum_{k=1}^{M-1} F_k^{n+1} \delta_h(x - X_k^{n+1}) \]

\[ t_k^{n+1} = \frac{D_s X_k^{n+1}}{|D_s X_k^{n+1}|} = \frac{X_{k+1}^{n+1} - X_k^{n+1}}{|X_{k+1}^{n+1} - X_k^{n+1}|} \quad k = 1, \ldots, M - 1. \]

\[ F_k^{n+1} = \gamma_3 D_s (t_k^{n+1}) \Delta s = \gamma_3 (t_{k+1}^{n+1} - t_k^{n+1}) \]

Discrete delta function \( \delta_h(x) = \delta_{1d}^h(x) \delta_{1d}^h(y), \)

\[ \delta_{1d}^h(x) = \begin{cases} \frac{1}{2h} (1 + \cos(\frac{\pi x}{2h})), & |x| \leq 2h, \\ 0, & \text{otherwise}. \end{cases} \]

Near the non-periodic boundary \( x = 0: \)

\[ \delta_{1d}^h(x - \bar{x}) = \delta_{h}^{hat}(x - 2h) + \frac{|\bar{x} - 2h|}{h} (\delta_{h}^{hat}(x - 2h) - \delta_{h}^{hat}(x - 3h)) \]

satisfies the first two moment conditions (Klinteberg et al 2014).
1st order projection method (Guermond et al, 2006):

\[
\begin{align*}
\rho \tilde{u}^{n+1} - \rho u^n &= \frac{3}{2} \rho (u^n \cdot \nabla)u^n - \frac{1}{2} \rho (u^{n-1} \cdot \nabla)u^{n-1} \\
&= - \nabla (2p^n - p^{n-1}) + \frac{1}{Re} \nabla \cdot (\eta D(\tilde{u}^{n+1})) + \frac{1}{Re} \nabla \cdot \tau^{n+1} + \frac{1}{We} f^{n+1} \\
- \beta u^{n+1} &= \eta l_s \partial_n u^{n+1} + l_s t \cdot \tau^{n+1} \cdot n, \quad v^{n+1} = 0. 
\end{align*}
\]

(15)

\[
\begin{align*}
\nabla (p^{n+1} - p^n) &= \frac{\rho}{\Delta t} (u^{n+1} - \tilde{u}^{n+1}) \\
\nabla \cdot u^{n+1} &= 0 \\
u^{n+1} \cdot n &= 0.
\end{align*}
\]

(16)
Equal density and equal viscosity; \( L_x \times L_y = 4 \times 1 \); static contact angle \( = 90^\circ \); \( Re = 1, l_s = 0.1, E_d = 50, \Delta t = 10^{-5} \).
Convergence Study

Different mesh sizes: \(1024 \times 256, 512 \times 128, 256 \times 64, 128 \times 32\) and \(64 \times 16\);
The numbers of markers: \(M = 304, 152, 76, 38, 19\), respectively.
Influence of Polymer on MCL

Capillary number $Ca = 0.07$, Weissenberg number $Wi = 0.1$, polymer zero-shear viscosity $\eta_p^1 = \eta_p^2 = \eta_p = 0.5$.

Mesh size = $256 \times 64$; number of markers $M = 76$. Steady interface profiles: less contact line slip for polymeric fluids (Dashed red) than for Newtonian fluids (Solid black).
Fix Capillary number $Ca = 0.07$; vary Weissenberg number $Wi = 0.01, 0.05, 0.1$ for $\eta_p = 0.5$; vary the zero-shear viscosity $\eta_p = 0.25, 0.5, 1.0$ for $Wi = 0.1$. Dominant factor: Polymer viscosity
Apparent contact angle measured at $Ca \ln l s^{-1}$ distance from the contact line;
Larger polymer density $\implies$ larger apparent contact angle $\implies$ less slip
No steady state for $\eta_p = 2.0$ and $Ca \geq 0.063$. 

![Graph showing Apparent Contact Angle vs. Capillary Number](image-url)
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Continuum models for MCL with insoluble surfactants are derived from the consideration of thermodynamics; a finite element method with front tracking techniques is developed.

The addition of surfactant will accelerate the contact line dynamics, and will change the equilibrium contact angle.

A sharp interface model with FENE-P assumption is proposed for MCL with additive polymers; a front tracking based immersed boundary method is developed.

Polymer viscosity is the dominant factor over fluid memory in moving contact line dynamics; the addition of polymer reduces slip near the contact line.
In an ongoing work, we are extending Cox’s asymptotic analysis for quasi-static MCLs to MCLs at finite time: the evolution of apparent contact angle and a relation between apparent contact angle and static contact angle.

- Perform 3D simulations. Level-set method developed by Dr Xu Shixin.
- Extend the model for insoluble surfactant to soluble surfactant.
- Improve the model for MCL with polymer with more solid mathematical and physical support.
Thank you!