Analysis on Weaker Forms of Compactness Via Grills

**Sketch of the talk:**

1. New classes of sets and their applications
2. $G$-compactness, $G$-Paracompactness and $G$-weak compactness
3. Nearly compactness and convergence of grills
4. Weaker forms of compactness

**Introduction and preliminaries**

1. Let $(X, \tau)$ be a topological space with no separation properties assumed. A sub collection $G$ (not containing the empty set) of $P(X)$ is called a grill [4] on $X$ if $G$ satisfies the following conditions:
   1. $A \in G$ and $A \subseteq B$ implies that $B \in G$,
   2. $A, B \subseteq X$ and $A \cup B \in G$ implies that $A \in G$ or $B \in G$.

2. Let $(X, \tau)$ be a topological space and $G$ be a grill on $X$. A mapping $\Phi: P(X) \rightarrow P(X)$ is defined as follows: $\Phi(A) = \Phi_G(A, \tau) = \{x \in X : A \cap U \in G \text{ for all } U \in \tau(x)\}$ for each $A \in P(X)$. The mapping $\Phi$ is called the operator associated with the grill $G$ and the topology $\tau$.

3. Let $G$ be a grill on a space $X$. Then a mapping $\Psi: P(X) \rightarrow P(X)$ is defined by $\Psi(A) = A \cup \Phi(A)$ for all $A \in P(X)$ [15]. The map $\Psi$ is a Kuratowski closure axiom.

Corresponding to a grill $G$ on a topological space $(X, \tau)$, there exists a unique topology $\tau_G$ on $X$ given by $\tau_G = \{U \subseteq X : \Psi(X - U) = X - U\}$, where for any $A \subseteq X$, $\Psi(A) = A \cup \Phi(A) = \tau_G - c l(A)$. An operator $\Gamma_\tau$ is defined as $\Gamma_\tau(A) = X - \Phi(X - A)$.

I. New classes of sets and their applications

We introduce three classes of sets namely $A_{\xi}$ set, $G$ gb-closed set and fuzzy $G$ gp-closed sets. Further $G$ fgp-normal, b-normal, $G$ gp regular and b-regular spaces are introduced. Let $(X, \tau, G)$ be a grill topological space. A subset $A$ of $X$ is called $A_{\xi}$ set, If $A \subseteq \text{int}(\psi(\Gamma_\tau(A)))$. The collection of all $A_{\xi}$ sets in $(X, \tau, G)$ is denoted by $\mathcal{A}_{\xi}(X, \tau, G)$. Then we define continuous and contra-continuous functions. If $A$ is $\tau_\xi$-dense-in-itself and $G_{\xi}$-closed set in a fuzzy $G$ -space $(X, \tau, G)$, then $A$ is gf-closed.

II. $G$-compactness, $G$-Paracompactness and $G$-weak compactness

$G$ -compactness, $G$ -paracompactness and $G$ -weak compactness induced by $\theta$ open sets. One point compactification is also done with the help of the new closure operator $cl_\theta$ satisfying the kuratowski’s closure axioms. Let $(X, \tau)$ be a non- $\theta$-compact, locally compact, $\theta$-T$_2$ space. By adjoining a new point $p \notin X$ to $X$, one can construct an extension space $X^* = X \cup \{p\}$ having the following properties: (1) $X^*$ is $\theta$-Hausdorff, (2) $X$ is dense in $X^*$, then $X^*$ is $\theta$-compact. We attempt to find an analogue of Alexander’s $\theta$-subbase theorem for $G$ - $\theta$-compactness. The E.Micheal’s theorem for $\theta$-open sets is also modified. $X$ is said to be $\theta$-
paracompact with respect to the grill or simply $G$-$\theta$ paracompact if every $\theta$-open cover $U = \{U_\alpha : \alpha \in \lambda\}$ of $X$ has a precise locally finite $\theta$-open refinement $U^*$ such that $X \setminus \cup U^* \notin G$.

### III. Nearly compactness and convergence of grills

The concept of $\delta$-convergence and $\delta$-adherence of grills is introduced and its various properties are discussed. We learn about the relationships among $G$-nearly compact spaces, $G$-totally cocompact spaces and almost $G$-cocompact spaces. Further we introduce the concept of relative grill with respect to the subspace $(A, \tau_\mu)$ of the space $(X, \tau, G)$ and study its properties. We also introduce a new space called AS$_\mu$ space and device its characterizations. The space $X$ is said to be nearly compact space if for every open cover $V = \{U_\alpha : \alpha \in \Lambda\}$ of $X$, there exists a finite subcollection such that $X \setminus \bigcup_{i=1}^n \text{intcl} U_i \notin G$.

### IV. Weaker forms of compactness

This part of the talk contemplates the study of other weaker forms of compact spaces like co-compact spaces, Co-paracompact spaces, Lindelof spaces, n-star compact spaces and finally almost paracompact spaces with respect to the grills. Finite intersection property with respect to the grills is introduced, as a tool to investigate the properties of compactness. A collection $\xi = \{A_\alpha : \alpha \in \Lambda\}$ of sets has the finite intersection property with respect to grill or simply $G$.f.i.p provided that the intersection of any finite subcollection belongs to grill. That is $\bigcap_{i=1}^n A_i \in G$. Results like A Ic-closed subset of an $G$-Cocompact space is $G$-Cocompact are obtained. A space $X$ is said to be n-star $G$-compactness if for every open cover $V$ of $X$, there is some finite subset $V$ of $U$ such that $X \setminus \text{st}(F, U) \notin G$, where $\text{st}(F, U) = \bigcup \{U \in U ; U \cap F \neq \emptyset\}$. Further we analyze the notions of n-star $G$-compactness and w-star-$G$-compactness. We establish the relationships among countably compact, strongly star compact and star compact with respect to grills.