Assume Admissible Synthesis

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REACTIVE SYNTHESIS

Classical setting

$\phi$ is the winning objective for $\text{Sys}$

$\text{Env}$ is adversarial

$\Rightarrow$ 2-player zero-sum game

Timing strategy = Correct $\text{Sys}$
→ Env is completely adversarial
?

what if Env = Rational user

→ Sys and/or Env can be made of several components, each with their own objective.

→ We need a richer setting
$N$-players turn-based graph games

Finite directed graph

Vertices are partitioned
$V = V_1 \cup V_2 \cup \ldots \cup V_N$

$V_i =$ vertices of Player $i$

$E \subseteq V \times V$
Players and objectives

\[ \mathcal{P} = \{1, 2, \ldots, N\} \]

Players stand for:
- parts of the system to design
- parts of the environment

\[ \Phi_1 \parallel \Phi_2 \parallel \cdots \parallel \Phi_N \]

\[ \phi_i \in \mathcal{V}^* \]

Strategies

\[ \sigma_i : \mathcal{V}^* \mathcal{V}_i \rightarrow \mathcal{V} \]

s.t. \[ \forall \tau, \sigma \in \mathcal{V}^* \mathcal{V}_i : \]
\[ (\sigma, \sigma_\tau(\tau \cdot \sigma)) \in E \]

\[ \mathcal{E}_i = \text{set of strategies of player } i. \]
Strategy profiles and outcomes

\[(\sigma_1, \sigma_2, \ldots, \sigma_n) \in \Xi_1 \times \Xi_2 \times \cdots \times \Xi_n \]

profile \[= (\sigma_i, \sigma_{-i})\]

\[\text{Out}_i (\sigma_1, \sigma_2, \ldots, \sigma_n) = \pi_0 \sigma_1 \sigma_2 \cdots \sigma_n \cdots = T\]

\[\text{st. : } \pi_0 = \sigma\]

\[\forall j > 0 : \text{ if } \pi(j) \in V_i \text{ then } \pi(j+1) = \sigma_i(\pi(0,j))\]
Running example

2 players

\[
\phi_1 \equiv \Box \Diamond 4 \\
\phi_2 \equiv \Box \Diamond 3
\]

- \( \Box \) = Player 1
- \( \Box \) = Player 2
SYNTHESIS RULES
Synthesis rules

\[
\exists \sigma_1 \cdot \forall \sigma_2 \ldots \forall \sigma_N : \\
\Theta_{\mathcal{W}}(\sigma_1, \sigma_2, \ldots, \sigma_N) = \phi_1
\]

= 2-player zero-sum case

\% Can be instantiated for any Players.
None of the players has a winning strategy!
Running example

By playing $1 \rightarrow 5$ Player 1 spoils $\phi_2 \equiv \square \diamond 3$
Running example

By playing $2 \rightarrow 2$, Player 2 spoils $\varphi_1 \equiv \Box \Diamond 4$

\(\varphi_1 = \Box \Diamond 4\)
\(\varphi_2 = \Box \Diamond 3\)
Synthesis rules

\[ \exists \sigma_1 \cdot \forall \sigma_2 \ldots \cdot \forall \sigma_N : \]
\[ \text{Our}(\sigma_1, \sigma_2, \ldots, \sigma_N) = \phi_2 \land \ldots \land \phi_N \rightarrow \phi_1 \]

Can be instantiated for any Players.
Running example

Player 1 wins $\phi_2 \rightarrow \phi_1$ with $1 \rightarrow 5$

$\phi_1 = \square \Diamond 4$

$\phi_2 = \square \Diamond 3$

$\Rightarrow$ useless solution!
ADMISSIBILITY
Dominated strategy

\( \sigma_i \) is dominated by \( \sigma_i' \) for \( \phi_i \) if
Dominated strategy

\[ \sigma_i \text{ is dominated by } \sigma'_i \text{ for } \phi_i \text{ if } \]

\[ \forall \sigma_{-i} \in \mathcal{X}_{-i}: \]

\[ \Omega^R(\sigma_i, \sigma_{-i}) = \phi_i \Rightarrow \Omega^R(\sigma'_i, \sigma_{-i}) = \phi_i \]

% Always as good
Dominated strategy

\[ \sigma_i \text{ is dominated by } \sigma'_i \text{ for } \phi_i \text{ if } \]

1. \( \forall \sigma_{-i} \in \mathcal{Z}_{-i} : \quad \Theta_{\mathcal{W}}(\sigma_i, \sigma_{-i}) = \phi_i \Rightarrow \Theta_{\mathcal{W}}(\sigma'_i, \sigma_{-i}) = \phi_i \quad \% \text{ Always as good} \)

2. \( \exists \sigma_{-i} \in \mathcal{Z}_{-i} : \quad \Theta_{\mathcal{W}}(\sigma_i, \sigma_{-i}) \neq \phi_i \land \Theta_{\mathcal{W}}(\sigma'_i, \sigma_{-i}) = \phi_i \quad \% \text{ Sometimes better} \)
Dominated strategy

\( \sigma_i \) is dominated by \( \sigma'_i \) for \( \phi_i \) if

1. \( \forall \sigma_{-i} \in \Xi_{-i} : \quad \Omega_{\mathcal{R}}(\sigma_i, \sigma_{-i}) = \phi_i \Rightarrow \Omega_{\mathcal{R}}(\sigma'_i, \sigma_{-i}) \neq \phi_i \)

% Always as good

2. \( \exists \sigma_{-i} \in \Xi_{-i} : \quad \Omega_{\mathcal{R}}(\sigma_i, \sigma_{-i}) \neq \phi_i \land \Omega_{\mathcal{R}}(\sigma'_i, \sigma_{-i}) = \phi_i \)

% Sometimes better

\[ \rightarrow \text{ A rational player avoids dominated strategies} \]
Any strategy that takes $1 \rightarrow 5$ is dominated by the strategy $1 \rightarrow 2, 3 \rightarrow 4$, even if it is not a winning strategy.
Admissible strategy

- $\sigma_i \in \Xi_i$ is admissible for $\phi_i$ if $\sigma_i$ is not dominated by any $\sigma_i' \in \Xi_i$ for $\phi_i$.

- $\text{Adm}_i (\phi_i) \subseteq \Xi_i$ is the set of admissible strategies of Player $i$ for $\phi_i$.

- $\text{Adm}_i (\phi_i) = \text{the only } \{ \text{rational} \} \text{ strategies}$.

- $\text{Adm}_i (\phi_i) \neq \emptyset$, $\forall \phi_i$. 
\[ \phi_2 = 4 \]

\( \sigma_2 \) is admissible

\[ \sigma_1 \]

\( \sigma_1' \) odd visits

\( \sigma_1' \) even
Synthesis rules

Assume Admissible

\[ s_1 \parallel s_2 \parallel \ldots \parallel s_N \]

\[ \phi_1 \quad \phi_2 \quad \phi_N \]

\[ \exists (\sigma_1, \sigma_2, \ldots, \sigma_N) : \]

1. \[ \sigma_i \in \text{Adm}(\phi_i) \text{ for all } i, 1 \leq i \leq N. \]

2. \[ \forall \sigma_{-i} \in \text{Adm}_{-i}(\phi_{-i}) : \text{Out}(\sigma_i, \sigma_{-i}) = \phi_i. \]

Each \( \sigma_i \) is winning against all admissible strategies of the others.
Claim: $$\forall \sigma_2 \in \text{Adm}_2(\phi_2) \cdot \forall \sigma_1 \in \text{Adm}_1(\phi_1):$$

$$\Theta_{\sigma_2}(\sigma_2, \sigma_2) = \phi_1 \wedge \phi_2$$

$$\Rightarrow \text{Assume Admissible rule applies!}$$
Assume Admissible Synthesis

**Theorem:** for all AA-profiles \((\sigma_1, \sigma_2, \ldots, \sigma_n)\):
\[
\Omega_R(\sigma_1, \sigma_2, \ldots, \sigma_n) = \phi_1 \land \phi_2 \land \ldots \land \phi_n
\]

**Theorem:** the set of AA-profiles is rectangular.
\[
= ST_1 \times ST_2 \times \ldots \times ST_n
\]
\[
ST_i = \{ \sigma_i \in \text{Adm}(\phi_i) \mid \forall \sigma_{-i} \in \text{Adm}_{-i}(\phi_{-i}): \Omega_R(\sigma_i, \sigma_{-i}) = \phi_i \}
\]

\[\Rightarrow \text{No need for synchronization!}\]
\[\Rightarrow \text{Compositionality}\]
Theorem: deciding if AA applies is Pspace-C for Safety, Reachability and Muller objectives.
**Theorem:**

Deciding if AA applies is Pspace-C for Safety, Reachability and Muller objectives.

State Space

\[
\begin{array}{c|c|c}
\text{Value 1} & \text{Value 0} & \text{Value -1} \\
\begin{array}{c}
\llbracket 2 \rrbracket \square \text{Safe} \\
\end{array} & \begin{array}{c}
\llbracket 1,2 \rrbracket \square \text{Safe} \\
\end{array} & \begin{array}{c}
\llbracket 1,2 \rrbracket \square \neg \text{Safe} \\
\end{array}
\end{array}
\]

\[\phi_2 = \square \text{Safe}\]

A player when playing admissible never decrease its value.
**Theorem:** deciding if AA applies is \( \text{Pspace-C} \) for Safety, Reachability and Muller objectives.

A player when playing admissible never decrease its value.

For other properties:

\begin{align*}
\text{Value}_1 & \Rightarrow \text{Win!} \\
\text{Value}_0 & \Rightarrow \text{Win a Help!} \\
\end{align*}

\[ \Rightarrow \text{The set of plays compatible with admissible strategies is \( \omega \)-regular} \]
Conclusion

- Assume admissible synthesis allows for compositional synthesis in $\mathcal{N}$-player non-zero sum games.
- Rectangular sets of solutions.
- If win gives a solution then AA gives a solution.
- If win-hyp gives a solution and $\langle \langle 2 \rangle \rangle \neq_2$ then AA gives a solution.