Let $c : [\omega]^2 \to r$. A monochromatic path of color $j$ is a listing (possibly empty) of integers $\{a_0, a_1, a_2 \ldots\}$ such that, for all $i \geq 0$, if $a_{i+1}$ exists then $c(\{a_i, a_{i+1}\}) = j$. A empty listing can be a path of any color. A singleton can be a path of any color. Paths might be finite or infinite. The color is determined for paths of more than one node. Improving on a result of Edrös, in 1978, Rado published a theorem which implies:

**Theorem 1** (Rado). Let $c : [\omega]^2 \to r$. Then, for each $j < r$, there is a monochromatic path of color $j$ such that these $r$ paths (as sets) partition $\omega$ (so they are pairwise disjoint sets and their union is everything).

We will provide some results and proofs which allow us to partially analyze the effective content of this theorem. These results and proofs are work with Greg Igusa, Ludovic Patey, and Mariya Soskova.