FSI Analysis of Diseased Coronary using Patient Specific Data

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A vulnerable carotid artery plaque exhibiting calcification (white) and intraplaque hemorrhage (purple) in addition to a lipid core (yellow)

Histology: Plaque with a Ruptured Cap

Photo of a Carotid Plaque
1. Diabetes and hypertension may evolve into cerebrovascular (stroke) or cardiovascular (heart attack) diseases.

2. Calcium and lipid enter vessel wall.

3. The material properties of vessel are changed and will cause stenosis (make some part of arteries become very narrow).

4. Hypertension (or sudden changes of flow conditions) may cause larger stress/strain at FS interface or inner part of vessel wall.

A transverse view of a diseased artery (post-processed)
X-Ray Image of Coronary

(a) Angiographic image showing location of the imaged coronary segment.

(b) Enlarged view of the segment and flow direction.

(c) Illustration of coronary vessel shape.

Fig. X-Ray angiographic images were used to determine location of the myocardium and the curvature of the coronary tube. Flow direction is marked in (b).
Why IVUS images? **IVUS images have much better resolution**. (25 micron in-plane resolution from IVUS vs. 300 micron from in vivo MRI).

IVUS data acquisition (the camera is rotating).
The Fluid Structure Interaction Models

\[
\begin{align*}
\begin{cases}
\rho_f \frac{\partial u}{\partial t} + \rho_f \text{div}(u \otimes u) - \text{div} \sigma_f(u, p) &= f^f \\
\text{div} u &= 0
\end{cases}
\end{align*}
\]
in \(\Omega_t^f(\eta)\),

\[
\rho_s^0 \frac{\partial^2 \eta}{\partial t^2} - \text{div}_\xi \sigma_s^0(\eta) = f_0^s,
\]
in \(\Omega_0^s\),

\[
u(t, x) = \frac{\partial \eta}{\partial t}(t, \xi), \quad \text{with} \ x = \mathcal{L}_t(\xi)
\]
on \(\Gamma_t\)

\[
\sigma_f(u, p)n^f = -\sigma_s(\eta)n^s.
\]
on \(\Gamma_t\)

Linear elasticity:

\[
\sigma_s^0(\eta) = \lambda \text{div}(\eta)I + 2\mu D(\eta)
\]
Brief Introduction of Structure Mechanics

Lagrangian map: \( \mathbf{x} = \mathbf{x}(\xi, t) = \mathcal{L}_t(\xi) \)

- \( \xi \): coordinate of material point in reference configuration
- \( \mathbf{x} \): coordinate of material point in deformed configuration
- \( \mathbf{\eta}(\xi, t) = \mathbf{x} - \xi \): displacement of material point

Lagrangian frame (mostly used for solids)

Kinematics is described in the reference configuration:

- **Position**: \( \mathbf{x} = \mathbf{x}(\xi, t) \)
- **Velocity**: \( \mathbf{u} = \mathbf{u}(\xi, t) = \dot{\mathbf{x}}(\xi, t) \)
- **Acceleration**: \( \mathbf{a} = \mathbf{a}(\xi, t) = \ddot{\mathbf{x}}(\xi, t) \)
Brief Introduction of Structure Mechanics...

Let us introduce the deformation gradient tensor

\[
F = \nabla_\xi x = I + \nabla_\xi \eta \\
F_{ij} = \frac{\partial x_i}{\partial \xi_j}
\]

Given an infinitesimal material line segment \(d\xi\) in \(\Omega^\xi_0\), it will be transformed through the Lagrangian map into \(dx = Fd\xi\).

- **Right Cauchy-Green tensor** \(C = F^T F\)

Constitutive relations – Green elastic materials

For a Green elastic material (also called hyperelastic) the stress tensor is derived from a strain energy function \(W\)

\[
\sigma^0_s = \frac{\partial W}{\partial F} \\
\rightarrow (\sigma^0_s)_{ij} = \frac{\partial W}{\partial F_{ij}}
\]
An anisotropic Mooney-Rivlin model with parameters determined by bi-axial experiments:

\[
W = c_1(I_1 - 3) + c_2(I_2 - 3) + D_1[\exp(D_2(I_1 - 3)) - 1] + \frac{K_1}{2K_2}[\exp(K_2(I_4 - 1)^2) - 1],
\]

where

\[
I_1 = \sum C_{ii} = \text{tr}(C), \quad I_2 = \frac{1}{2}[\text{tr}(C)^2 - \text{tr}(C^2)], \quad I_4 = \sum C_{ij} n_i n_j,
\]

\(n = (0, \cos(\alpha), \sin(\alpha))\) is a unit vector with \(\alpha\) being the fibre angle. The material constants are determined by experiments,

\[
c_1 = -1312.9\text{kPa}, \quad c_2 = 114.7\text{kPa}, \quad D_1 = 629.7\text{kPa}, \quad D_2 = 2.0, \quad K_1 = 35.9\text{kPa}, \quad K_2 = 23.5.
\]
ALE formulation (assuming known the domain deformation)

The moving domain is recast at each time $t$ to a fixed configuration $\Omega_0^f$ through the ALE mapping $A_t$:

$$A_t : \Omega_0^f \rightarrow \Omega_t^f,$$

$$x(\xi, t) = A_t(\xi)$$

- domain velocity
  $$\mathbf{w}(x, t) = \frac{\partial A_t}{\partial t} \circ A_t^{-1}(x)$$

- ALE derivative
  $$\frac{\partial \mathbf{u}}{\partial t} \bigg|_\xi = \frac{\partial \mathbf{u}}{\partial t} \bigg|_x + \mathbf{w} \cdot \nabla_x \mathbf{u}$$

- Euler expansion
  $$\frac{\partial J_{A_t}}{\partial t} \bigg|_\xi = J_{A_t} \text{ div } \mathbf{w}, \quad J_{A_t} = \det(\nabla A_t)$$

- ALE Transport formula
  $$\frac{d}{dt} \int_{\Omega_t} \mathbf{u}(x, t) = \int_{\Omega_t} \left[ \frac{\partial \mathbf{u}}{\partial t} \bigg|_\xi + \mathbf{u} \text{ div } \mathbf{w} \right], \forall \Omega_t \subset \Omega_t^f$$
ALE Form of NS Equations

\[
\begin{align*}
\left. \frac{\partial \mathbf{u}}{\partial t} \right|_{\xi} + \rho_f ((\mathbf{u} - \mathbf{w}) \cdot \nabla) \mathbf{u} - \text{div } \sigma_f (\mathbf{u}, p) &= \mathbf{f} \\
\text{div } \mathbf{u} &= 0
\end{align*}
\text{ in } \Omega_t
\]

At the FS interface:

Velocity continuity.

Force balance.
Numerical Algorithms: Decoupled Approach

Given the velocity, pressure, displacement, the velocity of structure and the domain information at a time:

1. Extrapolate the displacement and the velocity of the interface.
2. Compute the ALE mapping (=update domain shape).
3. Solve the fluid model (ALE form) with Dirichlet boundary condition.
4. Solve the structure model with Neumann boundary condition.
5. Repeat 3 to 4 until the interface condition are satisfied, then go to next time step.

Commercial softwares: ANSYS, ADINA… Academic codes: LifeV, FEBio
Velocity and pressure profile at the entry of the coronary tube.

Fluid BCs:
\[ \frac{\partial \mathbf{u}}{\partial n} \bigg|_{\text{inlet, outlet}} = 0, \]
\[ p \bigg|_{\text{inlet}} = p_{\text{in}}(t), \quad p \bigg|_{\text{outlet}} = p_{\text{out}}(t) \]
Structure BCs: Fixing the two ends, other parts may have cycle bending.
Prescribed pressure conditions (an illustration). A simplified pressure profile for human coronary artery was scaled to 75-108 mmHg and used as the upstream pressure (Pin). Downstream pressure was chosen so that flow rate was within physiological range (34+6.3 mLiter/m).
IVUS Data Processing

(a) Selected IVUS slices from 45-slice set

(b) Contour plots of selected IVUS slices from automated APIA segmentation.

(c) Contour plots of selected IVUS slices after smoothing and used in FE model construction

(d) Enlarged view

(e) Enlarged contour

(f) Enlarged contour after smoothing

(g) 3D geometry showing 45 slices and lipid cores.
1. 2D slices need to be rotated as the camera is rotating (adjusted according to medical doctors’ suggestions).

2. Small calcified or lipid pieces are discarded or merged to neighbored larger pieces. The calcified/lipid contours are approximated by Spline curves and the partitioned into trapezoids.

3. In the curved 3D reconstruction, we use the X-Ray image to determine the curvature and the coordinates of the centers of each slices, then map the 2D slices into a 3D geometry.

4. The geometry may need to be shrinked so that the computational results match the IVUS images. (Lumen shrinks around 8-10 percent, but the wall boundary shrinks 1.6-2.3 percent to make sure the volume is conserved).
Mesh Generation

(a) Slice 1 Showing Component-Fitting Curves and “Surfaces”.

(b) Slice 2 Showing Component-Fitting Curves and “Surfaces”.

(c) Stacking Slices to Form Component-Fitting Volumes.

(d) Four types of volumes.

Hexahedron
Prism
Pyramid
Tetrahedron
Plots of Stress-$P_1$, Strain-$P_1$, flow velocity and flow maximum shear stress from the model with higher pressure conditions. Inlet pressure $P_{\text{max}}=150$ mmHg, $P_{\text{min}} = 80$ mmHg. Outlet pressure was adjusted accordingly.
Plot of Stress and Strain from One Sample

(a) Input contour.  (b) Computational contour.  (c) Stress P1.  (d) Strain P1.

Comparisons of contours, stress and strain distributions of a selected slice
1. Mathematical models and numerical algorithms for FSI based bio-mechanical analysis are presented.

2. Mesh generation, data acquisition and physiological flow simulation techniques are presented.

3. The stress/strain variations maybe affected by material properties, flow condition including strong pulsating pressure, shape and size of the calcified/lipid part, and very severe stenosis. (more works need to be done to justify/quantify).
Conclusions

The proposed approach provides an opportunity to improve current coronary plaque models with

a) better image resolution (25 micron in-plane resolution from IVUS vs. 300 micron from in vivo MRI),

b) actual measured blood pressure at the site of the lesion;

c) patient-specific vessel curvature;

d) biaxial mechanical testing data from a human coronary.
Thank You !