An arbitrary lagrangian eulerian discontinuous galerkin approach to fluid-structure interaction and its application to cardiovascular problem

Yifan Wang

University of Houston, Department of Mathematics
Joint work with: S. Canic and A. Quaini
Acknowledgements: NSF DMS-1263572, DMS-1262385.
Outline

- Motivation
- Mathematical Model for Applications in Hemodynamics
- ALE Discontinuous Galerkin Approach
- Numerical Results
- Conclusions and Future Study
Main motivation

We are interested in studying the blood flow inside the Abdominal Aortic Aneurysm with stent-graft treated.

- Provide new information and give insight into new treatment
- Serve as a potential predictor of AAA arterial wall rupture

Why Discontinuous Galerkin?

- High order and less dissipative solution
- Ideal for problems containing discontinuous solution
We treat blood as an *incompressible, viscous Newtonian* fluid, and its motion is described by the Navier-Stokes equations.

\[
\begin{align*}
\partial_t u + (u \cdot \nabla) u &= \frac{1}{\rho} (\nabla \cdot \sigma) = -\frac{1}{\rho} \nabla p + \nu \Delta u \\
\nabla \cdot u &= 0
\end{align*}
\]

in \( \Omega(t) \),

where \( u \) is the fluid velocity, \( \sigma \) is the Cauchy stress tensor, \( \rho \) is the fluid density, and \( \nu \) is the fluid kinematic viscosity. The Cauchy stress tensor is given by \( \sigma = -p I + 2\mu D(u) \), where \( p \) is the pressure and \( D(u) = (\nabla u + (\nabla u)^T)/2 \) is the strain rate tensor.
The elastodynamics of the thin structure will be described in general by

\[ \rho_s h_s \partial_{tt} \mathbf{\eta} + \mathbf{L} \mathbf{\eta} + \gamma \mathbf{V} \partial_t \mathbf{\eta} = \mathbf{f}_s \quad \text{in } \Gamma_0, \]

Operator \( \mathbf{L} \) is a linear operator acting on vector function \( \mathbf{\eta} \) describing the elastic properties of the structure, and \( \mathbf{V} \) is a linear operator acting on vector function \( \partial_t \mathbf{\eta} \) describing the viscoelastic properties of the structure. \( \Gamma_0 \) denote the fixed reference structure domain, \( \rho_s \) is the structure density, \( h_s \) is the structure thickness, \( \gamma \) is a viscoelastic parameter, and \( \mathbf{f}_s \) is the force acting on the structure.

We consider two different forms of operators \( \mathbf{L} \) and \( \mathbf{V} \).
Thin structure model - I

We consider the **generalized string model** for an elastic tube with the reference configuration as a cylinder of radius $R_0$ and length $L$ (Quarteroni 2000). And assuming that the longitudinal displacement $\eta_x$ and the azimuthal displacement $\eta_z$ are negligible.

\[
\mathcal{L}\eta = \begin{pmatrix}
0, -kGh_s \frac{\partial^2 \eta_y}{\partial x^2} + \frac{Eh_s}{(1 - \nu_s^2)} \frac{\eta_y}{R_0^2}, 0
\end{pmatrix}^T, \quad \mathcal{V} \frac{\partial \eta}{\partial t} = \begin{pmatrix}
0, -\frac{\partial^3 \eta_y}{\partial x^2 \partial t}, 0
\end{pmatrix}^T,
\]

where $k$ is the Timoshenko shear correction factor, $G$ is the shear modulus, $E$ is the Young’s modulus and $\nu_s$ is the Poisson ratio of the structure. Structure model is endowed with **absorbing boundary conditions** of first order:

\[
\frac{\partial \eta_y}{\partial t} - \sqrt{\frac{kG}{\rho_s}} \frac{\partial \eta_y}{\partial x} = 0 \quad \text{at } x = 0,
\]

\[
\frac{\partial \eta_y}{\partial t} + \sqrt{\frac{kG}{\rho_s}} \frac{\partial \eta_y}{\partial x} = 0 \quad \text{at } x = L.
\]
The second model we consider involves no spatial derivatives and no viscoelasticity. Operators $\mathcal{L}$ and $\mathcal{V}$ are given by:

$$\mathcal{L}\eta = (C_x \eta_x, C_y \eta_y, 0)^T,$$

$$\mathcal{V} = (0, 0, 0)^T,$$

where $C_x$ and $C_y$ are spring constants. Notice that the displacement components are independent. The structure model is supplemented with homogeneous Dirichlet boundary conditions:

$$\eta = 0 \quad \text{on } \partial \Gamma_0.$$
The coupling conditions

The fluid and the structure are coupled via two boundary conditions:

- **Kinematic coupling condition**: it describes the continuity of velocity at the fluid-structure interface (no-slip condition)

\[ \mathbf{u} \circ A_t = \partial_t \eta \quad \text{on } \Gamma_0 \times (0, T); \]

- **Dynamic coupling condition**: it describes the continuity of the stress at the fluid-structure interface

\[ J \, \hat{\sigma} \mathbf{n} \big|_{\Gamma(t)} = f_s \quad \text{on } \Gamma_0 \times (0, T), \]

where \( J \) denotes the Jacobian of the transformation from Eulerian to Lagrangian coordinates, and \( \hat{\sigma} \mathbf{n} \big|_{\Gamma(t)} \) denotes the normal fluid stress at the deformed fluid-structure interface, evaluated with respect to the reference configuration. Vector \( \mathbf{n} \) is the outward normal to the deformed fluid domain.
Coupling conditions above can be written in the equivalent form:

\[
\begin{align*}
\alpha_f \mathbf{u} \circ A_t - J \widehat{\mathbf{s}} \mathbf{n} |_{\Gamma(t)} &= \alpha_f \partial_t \eta - \mathbf{f}_s \quad \text{on } \Gamma_0 \times (0, T), \\
\alpha_s \mathbf{u} \circ A_t + J \widehat{\mathbf{s}} \mathbf{n} |_{\Gamma(t)} &= \alpha_s \partial_t \eta + \mathbf{f}_s \quad \text{on } \Gamma_0 \times (0, T),
\end{align*}
\]

where \( \alpha_f > 0 \) and \( \alpha_s > 0 \) (\( \alpha_f \neq \alpha_s \)) are constants.
The FSI problem will be solved using two different partitioned strategies based on Domain Decomposition methods: the Dirichlet-Neumann (DN) and the Robin-Neumann (RN) algorithms. (A. Quarteroni and A. Valli 1999)

**The Dirichlet-Neumann:**
Dirichlet boundary conditions are imposed at the interface $\Gamma(t)$ for the fluid problem, whereas Neumann boundary conditions are supplemented for the structure problem. The DN algorithm iterates over these two sub-problems until meeting the convergence criteria for the interface displacement. When the density of structure has the same magnitude as fluid density, relaxation step on the structure displacement is required (Nobile 2001).

**The Robin-Neumann:**
When the FSI problem contains a closure structure, the Dirichlet-Neumann algorithms fails due to the fact that the coupling conditions are not implemented synchronously. Robin-Neumann algorithm always converges without any relaxation and its convergence is almost insensitive to the added-mass effect.
ALE mapping

We consider a smooth mapping:

\[ A_t : \Omega_0 \rightarrow \Omega(t), \]
\[ A_t : x_0 \mapsto x, \]

where \( x \) and \( x_0 \) are the coordinates in the physical domain \( \Omega(t) \) and the reference domain \( \Omega_0 \), respectively. The domain velocity \( w \) is given by

\[ w(t, \cdot) = \frac{dA_t}{dt}(t, A_t(t, \cdot)^{-1}). \]

The ALE time derivative of the fluid velocity is defined as:

\[ \partial_t u|_{x_0} = D_t u(t, A_t(x_0)) = \partial_t u(t, x) + w(t, x) \cdot \nabla u(t, x), \text{ for } x = A_t(x_0), \ x_0 \in \Omega_0. \]
ALE mapping

The incompressible Navier-Stokes equations in the ALE formulation:

\[
\begin{align*}
\partial_t \mathbf{u} |_{x_0} + (\mathbf{u} - \mathbf{w}) \cdot \nabla \mathbf{u} &= -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} \\
\nabla \cdot \mathbf{u} &= 0
\end{align*}
\]

in \( \Omega(t) \),
ALE mapping

We introduce a notation of $F$ stands for the nonlinear flux vector. Given the velocity components $u = (u, v)^T$ for $d = 2$ and $u = (u, v, w)^T$ for $d = 3$, $F$ is defined as:

$$F = \begin{bmatrix} u^2 & uv \\ vu & v^2 \end{bmatrix} \quad \text{if} \quad d = 2,$$

or

$$F = \begin{bmatrix} u^2 & uv & uw \\ vu & v^2 & vw \\ wu & wv & w^2 \end{bmatrix} \quad \text{if} \quad d = 3.$$

Let $J_A_t = \det(\frac{\partial x}{\partial x_0})$ be the Jacobian of the deformation gradient:

$$\partial_t J_A_t|_{x_0} = J_A_t \nabla \cdot w.$$

We can derive the fully conservative form of the NS equation in ALE:

$$\partial_t (J_A_t u)|_{x_0} + J_A_t \nabla \cdot (F - wu) = -\frac{J_A_t}{\rho} \nabla p + J_A_t \nu \Delta u.$$
Describing the ALE map in terms of the fluid domain displacement \( d(x_0, t) = A_t(x_0) - x_0 \). Let \( \eta \) be the displacement of the deformable boundary \( \Gamma(t) \). Inside \( \Omega_0 \) the displacement \( d \) is arbitrary: it can be any reasonable extension of \( \eta \) over \( \Omega_0 \). A classical choice is to consider a harmonic extension in the reference domain, that is:

\[
\Delta d = 0, \quad \text{in} \quad \Omega(t),
\]

with boundary conditions:

\[
d = \eta \quad \text{on} \quad \Gamma(t),
\]
\[
d = 0 \quad \text{on} \quad \partial \Omega(t)/\Gamma(t).
\]

The fluid domain velocity \( w \) is obtained by differentiation: \( w = \partial_t d \).
The second order semi-implicit splitting scheme is used to discretize NS equation (Karniadakis 1991). The NS equation in the reference domain $\Omega_0$ is splitted to three sub-equations.

**Step 1:** Using the Backward Difference Formula (BDF2) to obtain the first intermediate velocity field $J^n u_{k+\frac{1}{3}}$ by the explicit approximation of the convective component $\nabla \cdot (F - wu)$:

\[
\frac{(3J^n u_{k+\frac{1}{3}} - 4J^n u^n + J^{n-1}u^{n-1})}{2\Delta t} = -2J^n \nabla \cdot (F - wu)^n + J^{n-1} \nabla \cdot (F - wu)^{n-1}.
\]

**Step 2:** Known the $J^n u_{k+\frac{1}{3}}$, we solve for the second intermediate velocity field $J^n u_{k+\frac{2}{3}}$ by including the pressure gradient term:

\[
\frac{J^n u_{k+\frac{2}{3}} - J^n u_{k+\frac{1}{3}}}{3 \frac{2\Delta t}{\rho}} = -\nabla J^n p_{k+1}.
\]

We impose the divergence free requirement to $J^n u_{k+\frac{2}{3}}$,
Temporal discretization of the fluid problem

Taking the divergence of both sides of the equation:

\[
\frac{1}{\rho} \Delta J^n p_{k+1} = \frac{3}{2\Delta t} \nabla \cdot (J^n u_{k+\frac{1}{3}}),
\]

and solve for the pressure \( J^n p_{k+1} \). After obtaining the pressure field \( J^n p_{k+1} \), the second intermediate velocity field \( J^n u_{k+\frac{2}{3}} \) is updated from:

\[
J^n u_{k+\frac{2}{3}} = J^n u_{k+\frac{1}{3}} - \frac{2\Delta t}{3\rho} \nabla J^n p_{k+1}.
\]

**Step 3:** Known the second intermediate velocity field \( J^n u_{k+\frac{2}{3}} \), we implicitly integrate the viscous component \( \Delta u \) to solve a Helmholtz problem for \( J^n u_{k+1} \):

\[
\frac{J^n u_{k+1} - J^n u_{k+\frac{2}{3}}}{2\Delta t} = \nu J^n \Delta u_{k+1}.
\]
Discontinuous Galerkin method

Let us consider the local polynomial approximation of fluid velocity component of $u(r, s)$:

$$u(r, s) = \sum_{i=1}^{N} \hat{u}_i \Phi_i(r, s) = \sum_{j=1}^{N} u_j l_j(r, s),$$

Here, we are assuming that the number of nodes and number of modes are the same, and are equal to $N$. $\hat{u}_i$ is the expansion coefficient associating with the orthonormal modal base $\Phi_i(r, s)$, while $u_j$ is the value of $u(r, s)$ at grid point of $(r, s)_j$ and $l_j(r, s)$ is the corresponding nodal base. In the 2D case, we have $N = (n + 1)(n + 2)/2$ which represents the least number of terms required to have the completeness of expansion using the $n^{th}$ order polynomial.
Discontinuous Galerkin method

Modal bases: A set of orthonormal polynomial modes defined in standard isosceles right triangle \( \Omega_{st} \) (Gottlieb 1977, Dubiner 1991, Hesthaven 2008):

\[
\Phi_i(r, s) = \sqrt{2} P_k^{0,0}(2 \frac{1 + r}{1 - s} - 1) P_i^{(2k+1,0)}(s)(1 - s)^k,
\]

in which \( r \) and \( s \) are the coordinates of two short sides of an isosceles right triangle. Subscript \( i \) stands for the \( i^{th} \) mode of the basis functions, and \( i = (N + 1)k + l + 1 - \frac{k}{2}(k - 1), (k, l) \geq 0; k + l \leq N \). \( P_{k}^{(\alpha,\beta)}(x) \) is Jacobi polynomials.

Nodal bases: Lagrangian polynomials built on Fekete points (GKarniadakis 2005)
Discontinuous Galerkin method

Nodal bases are expressed in terms of modal bases to simplify the evaluation of the numerical integration. And the generalized Vandermonde matrix $V$ works as a role to connect the nodal bases with modal bases, and it is defined as:

$$V_{ij} = \Phi_j(r, s)_i.$$ 

Between modal $\Phi_j(r, s)$ and nodal $l_j(r, s)$ bases, one can be transformed from the other via Vandermonde matrix:

$$l(r, s) = (V^T)^{-1}\Phi(r, s).$$

In other words, nodal basis $l_i(r, s)$, it can be expressed as:

$$l_i(r, s) = \sum_{k=1}^{N}(V^T)^{-1}ik\Phi_k(r, s).$$
Discontinuous Galerkin method

In an standard isosceles triangle, the local mass matrix is:

\[ M_{ij} = (VV^T)_{ij}^{-1}. \]

And the local matrices for convective term are:

\[ S_{rij} = (MD_r)_{ij}, \quad S_{sij} = (MD_s)_{ij}. \]

where \( D_r \) and \( D_s \) can be computed from:

\[ D_r = V_r V^{-1}, \]
\[ D_s = V_s V^{-1}. \]

And \( V_r \) and \( V_s \) denote two Vandermonde matrices as below:

\[ V_{r_{ij}} = \left. \frac{d\Phi_j(r,s)}{dr} \right|_{(r,s)_i}, \]
\[ V_{s_{ij}} = \left. \frac{d\Phi_j(r,s)}{ds} \right|_{(r,s)_i}. \]
Spatial discretization of the fluid problem

**Step 1:** For the convective equation, we have:

\[
M_e \frac{3J^n u_{k+\frac{1}{3}} - 4J^n u^n + J^{n-1} u^{n-1}}{2\Delta t} = -2J^n S : (F - wu)^n + J^{n-1} S : (F - wu)^{n-1}
\]
\[
- 2J^n \int_{\partial E} n \cdot ((F - wu) - (F - wu)^*)^n l(x, y) dxdy
\]
\[
+ J^{n-1} \int_{\partial E} n \cdot ((F - wu) - (F - wu)^*)^{n-1} l(x, y) dxdy.
\]

Local Lax-Friedrichs flux is used and denoted by \((F - wu)^*\):

\[
(F - wu)^* = \frac{(F - wu)^+ + (F - wu)^- + \tau (u^+ - u^-)}{2},
\]

in which the coefficient \(\tau\) is the local maximum velocity magnitude.
Spatial discretization of the fluid problem

**Step 2:** For the pressure Poisson equation, we convert it into a system of first order equations (Cockburn and Shu 1998):

\[
\begin{cases}
\frac{J^n}{\rho} \nabla p_{k+1} = q \\
\nabla \cdot q = \frac{3}{2\Delta t} \nabla \cdot (J^n u_{k+\frac{1}{3}})
\end{cases}
\]

In each triangle \(E\), we discretize \((J^n p_{k+1}, q)\) as \(((J^n p_{k+1})_e, q_x, q_y)\) and approximate it by the \(n^{th}\) order basis function \(l(x, y)\) constructed on that triangle. Internal penalty fluxes as below are used:

\[
\begin{cases}
q^* = \frac{J^n}{\rho} \left( \nabla (p_{k+1})_e^+ + \nabla (p_{k+1})_e^- \right) - \tau \left[ n^- (p_{k+1})_e^- + n^+ (p_{k+1})_e^+ \right] \\
p^*_{k+1} = \frac{p_{k+1}^+ + p_{k+1}^-}{2}
\end{cases}
\]

where the \(\tau\) is penalty parameter.
Spatial discretization of the fluid problem

Then we have:

$$\frac{J^n}{\rho} \left( (S_x)_{epk+1} - \int_{\partial E} n_x (p_{k+1} - p^*_{k+1}) l(x, y) dxdy \right) = M_{eqx},$$

$$\frac{J^n}{\rho} \left( (S_y)_{epk+1} - \int_{\partial E} n_y (p_{k+1} - p^*_{k+1}) l(x, y) dxdy \right) = M_{eqy},$$

$$\begin{align*}
(S_x)_{eqx} & - \int_{\partial E} n \cdot (q_x - q^*) l(x, y) dxdy \\
+ (S_y)_{eqy} & - \int_{\partial E} n \cdot (q_y - q^*) l(x, y) dxdy = \frac{3}{2\Delta t} S \cdot (J^n u_{k+\frac{1}{3}}). 
\end{align*}$$
Consider the general string model for the structure, we introduce a new variable of $U$:

$$U = \rho_s h_s \frac{\partial \eta_y}{\partial t} - \gamma \frac{\partial^2 \eta_y}{\partial x^2},$$

which combines the temporal derivative terms. Such that the problem can be rewritten as:

$$\frac{\partial}{\partial t} \left( \rho_s h_s \frac{\partial \eta_y}{\partial t} - \gamma \frac{\partial^2 \eta_y}{\partial x^2} \right) = \frac{\partial U}{\partial t} = kG h_s \frac{\partial^2 \eta_y}{\partial x^2} - \frac{E h_s}{(1 - \nu^2)} \frac{\eta_y}{R^2_0} + J \hat{\sigma} n |\Gamma(t)|.$$

We also introduce new variables $W$ denote the second order spatial derivative term $\frac{\partial^2 \eta_y}{\partial x^2}$. 
Temporal discretization of the structure problem

As a result, the original problem is transformed into a system of ODEs:

\[
\begin{align*}
\rho_s h_s \frac{\partial \eta_y}{\partial t} & = U + \gamma W, \\
\frac{\partial U}{\partial t} & = k G h_s W - \frac{E h_s}{(1 - \nu^2)} \frac{\eta_y}{R^2_0} + J \hat{\sigma n} |_{\Gamma(t)}. 
\end{align*}
\]

The initial Conditions of \( \eta \) and \( U \) can be reasonably obtained as following:

\[
\begin{align*}
\eta_y |_{t=0} & = 0, \quad \frac{\partial \eta_y}{\partial t} |_{t=0} = 0, \\
U |_{t=0} & = \rho_s h_s \frac{\partial \eta_y}{\partial t} |_{t=0} - \gamma W |_{t=0} = 0.
\end{align*}
\]
Temporal discretization of the structure problem

The above system can be discretized by using forward difference scheme. Given $\eta^n_y$, $U^n$, $W^n$ and $\sigma_{k+1}$, find the value of $(\eta_y)_{k+1}$, $U_{k+1}$ and $W_{k+1}$, so that:

\[
\begin{align*}
\rho_s h_s \frac{(\eta_y)_{k+1} - \eta^n_y}{\Delta t} &= U_{k+1} + \gamma W^n, \\
\frac{U_{k+1} - U^n}{\Delta t} &= kGh_s W^n - \frac{Eh_s}{1 - \nu^2} \frac{\eta^n_y}{R_0^2} + J^n \overrightarrow{\sigma}_{k+1} \hat{n} |_{\Gamma(t^n)}, \\
W_{k+1} &= \frac{\partial^2 (\eta_y)_{k+1}}{\partial x^2}.
\end{align*}
\]
Spatial discretization of the structure problem

We introduce an intermediate variable $q$ to split $W$ into a system of two first order differential equations which can later be easily solved by Local Discontinuous Galerkin (LDG) (Cockburn and Shu 1998):

\[
\begin{aligned}
q &= \frac{\partial \eta_y}{\partial x} \\
W &= \frac{\partial q}{\partial x}.
\end{aligned}
\]

By choosing the fluxes as $q^* = q^-$ and $\eta_y^* = \eta_y^+$, we can write out the matrix form of eq. (1) as below:

\[
\begin{aligned}
M_eq &= (S_x)e\eta_y - \int_{\partial E} n_x \cdot (\eta_y - \eta_y^*)l(x, y)dxdy, \\
M_eW &= (S_x)eq - \int_{\partial E} n_x \cdot (q - q^*)l(x, y)dxdy;
\end{aligned}
\]
Element-wise ALE mapping

The **internal penalty flux** also is used in solving the displacement equation in the mapping, our approach does not force two neighbor triangles to have the same displacement at the common edge but limit the differences between them and disallow large gaps in the displacement.

We reconstruct harmonic extension equation as a system of two first order equations for each component of displacement vector:

\[
\left\{ \begin{array}{l}
\nabla d = q \\
\n\nabla \cdot q = 0
\end{array} \right.
\]

where \(d\) corresponds to each component of the displacement vector. We then replace \(q\) with the following internal penalty fluxes:

\[
q^* = \frac{\nabla d_e^+ + \nabla d_e^-}{2} - \tau [n^- d^- + n^+ d^+], \quad d^* = \frac{d_e^+ + d_e^-}{2}.
\]

Here, \(d\) correspond to the vector of each component of displacement. Parameter \(\tau\) is the penalty parameter.
Element-wise ALE mapping

Then the matrix form are:

\[
M_e q_x = (S_x)_e d - \int_{\partial E} n_x (d - d^*) l(x, y) dxdy,
\]

\[
M_e q_y = (S_y)_e d - \int_{\partial E} n_y (d - d^*) l(x, y) dxdy;
\]

\[
(S_x)_e q_x + (S_y)_e q_y - \int_{\partial E} n \cdot (q_x - q^*) l(x, y) dxdy
\]

\[
- \int_{\partial E} n \cdot (q_y - q^*) l(x, y) dxdy = 0.
\]
Benchmark test 1: Compliant vessel

(a) Computational domain

(b) Inlet data over time

(c) Mesh

Figure: Test 1: (a) computational domain at the initial time, (b) the inlet pressure pulse, and (c) unstructured triangle mesh (fine mesh).
Benchmark test 1: Compliant vessel

Figure: Test 1: pressure at different times: (a) $t = 2$ ms, (b) $t = 4$ ms, (c) $t = 6$ ms, (d) $t = 8$ ms, (e) $t = 10$ ms, (f) $t = 12$ ms. The unit for the pressure is dynes/cm$^2$. 
Benchmark test 1: Compliant vessel

Figure: Test 1: pressure along the centerline at different times: (a) $t = 2$ ms, (b) $t = 4$ ms, (c) $t = 6$ ms, (d) $t = 8$ ms, (e) $t = 10$ ms, (f) $t = 12$ ms. In the legend, A. Quaini denotes the results in (Quaini 2009), DG-IP_HI and DG-IP_LO denote the results obtained with our DG solver with IP fluxes with the fine and coarse mesh, respectively.
Benchmark test 2: Oscillating immersed linearly elastic 1D membrane

Figure: Test 3: (a) coarse mesh and (b) fine mesh.
Benchmark test 2: Oscillating immersed linearly elastic 1D membrane

(a) $t = 0.1$ s  
(b) $t = 0.5$ s  
(c) $t = 1$ s  
(d) $t = 1.5$ s  
(e) $t = 2$ s  
(f) $t = 8$ s

**Figure:** Test 3: pressure contour and velocity vector field at time (a) $t = 0.1$ s, (b) $t = 0.5$ s, (c) $t = 1$ s, (d) $t = 1.5$ s, (e) $t = 2$ s, (f) $t = 8$ s.
Benchmark test 2: Oscillating immersed linearly elastic 1D membrane

Figure: Test 3: (a) maximum x-coordinate over time, (b) difference in absolute value between the curves in (a).
Section of an Abdominal Aortic Aneurysm

We consider a section of a patient-specific descending aorta affected by AAA with compliant wall.

Figure: (a) Geometry of the section of aorta affected by AAA under consideration and (b) mesh used for the computations.
Figure: Rescaled (a) experimental average velocity ($cm^2/s$) at the inlet section and (b) experimental pressure ($Pa$) at the outlet section.
Introduction
Mathematical Model
ALE Discontinuous Galerkin Approach
Numerical Results
Conclusions and Future Study

Benchmark test 1
Benchmark test 2
Section of an Abdominal Aortic Aneurysm
Preliminary 3D simulation

(a) Pressure
(b) Velocity magnitude
(c) Displacement magnitude
Figure: (a) and (d) Pressure, (b) and (e) velocity magnitude and vector field, (c) and (f) displacement magnitude on a section of aorta affected by AAA without (top) and with (bottom) stent-graft at time $t = 2.3$ s.
Introduction
Mathematical Model
ALE Discontinuous Galerkin Approach
Numerical Results
Conclusions and Future Study

Benchmark test 1
Benchmark test 2
Section of an Abdominal Aortic Aneurysm
Preliminary 3D simulation

(a) Pressure
(b) Velocity
(c) Displacement
Figure: (a) and (d) Pressure, (b) and (e) velocity magnitude and vector field, (c) and (f) displacement magnitude on a section of aorta affected by AAA without (top) and with (bottom) stent-graft at time $t = 2.85$ s.
**Figure:** Maximum displacement magnitude over time (3 cycles): blue line corresponds the case without stent-graft, red line corresponds the case with the stent-graft.
Preliminary 3D simulation

(a) Inlet data over time

Figure: The inlet pressure pulse.
Conclusions and Future Study

- We proposed an DG ALE approach for FSI problem. Benefit from allowing an element-wise mapping, the implementation of DG in ALE frame becomes straightforward.
- For certain FSI which contains discontinuity solution our DG-ALE approach will be a feasible approach.
- Our solver is a serial version only at the moment, parallelization is desired.
Conclusions and Future Study

THANK YOU FOR YOUR ATTENTION!