Robust Preferences

and

Robust Portfolio Choice

Handbook "Mathematical Modeling and Numerical Methods in Finance"
Ed. Bensoussan & Zhang
Elsevier (2009)
Hans Föllmer
Humboldt University, Berlin

Alexander Schied
Cornell, ORIE (- 2008)
U. Mannheim (2009- )

Stefan Weber
Cornell, ORIE (- 2009)
U. Hannover (2009 - )
The Turner Review:

A regulatory response to the global banking crisis

(FSA = Financial Services Authority
March 2009)

1.1. (iv), p. 22:

"Misplaced reliance on sophisticated maths"
1.4 (iii), p. 44-45:

"mathematically modelable risk"

vs.

"Knightian uncertainty"

Frank Knight:
"Risk, Uncertainty, and Profit"
(Dissertation, Yale, 1916 / 21)
F. Knight's distinction:

"Risk": we have a probabilistic model \((\Omega, \mathcal{F}, P)\)

possible scenarios \(\Omega\)

probability measure on possible events \(P\)

"Uncertainty": we don't know \(P\)

i.e., model - uncertainty

ambiguity

risk
How (in principle) can we come up with a P?

i) Looking *backward* (and hoping for some stationarity...) using empirical distribution ("data")

ii) Looking *forward* in a "rational" manner (cf. Savage below)

Ambiguity! under constraints
How can we deal (mathematically!) with model uncertainty / ambiguity?

"Robustness":

a case study:
"Robust Preferences"
What are "reasonable" preferences on \( \mathcal{X} \)?

\( \mathcal{X} \) is a set of possible choices.

In our financial context:

\( \mathcal{X} \) is a linear space of "financial positions".

\[ X : \Omega \rightarrow \mathbb{R}^1 \]

\( \omega \rightarrow X(\omega) \)

"\( X(\omega) \) is the monetary (discounted) outcome at the end of trading period in scenario \( \omega \)."
Typically:

\[ X > Y \iff U(X) > U(Y) \]

for some "numerical representation"
\[ U : \mathcal{X} \rightarrow \mathbb{R}^+ \]

classical paradigm of "expected utility":

\[ U(X) = E_p [u(X)] \]

for some implicit

- utility function \( u \)
  (↑, usually concave, but not always; cf. Behavioral Finance)
- probability measure \( P \) on \( (\Omega, \mathcal{F}) \)
Note: \[ U(x) = E_P[u(x)] = \int \mu_x \]

\[ \mu_x := \text{law of } x \text{ under } P \]

can be viewed as a functional on "lotteries" \( u \in \mathcal{CL}( \mathbb{R}^1) \), if \( P \) is already fixed (von Neumann-Morgenstern)

\[ \iff \]

von Neumann-Morgenstern,
Savage

classical axioms of "rationality"

in particular:

"independence axiom":

\[ a > b \iff \alpha a + (1-\alpha) b > \alpha b + (1-\alpha) a \]

for any \( A \), any \( \alpha \in (0,1) \)
extended setting (Aumann, ...):

\[ \hat{X} \subseteq \hat{X} \]

= all \( X : \Omega \rightarrow \mathbb{M}(\mathbb{R}) \)

\[ \omega \rightarrow \hat{X}(\omega, \cdot) \]

"lottery"

- contains both
cotteries:
financial positions:

\[ \hat{X}(\omega, \cdot) \equiv \mu \]

\[ \hat{X}(\omega, \cdot) = S \hat{X}(\omega) \]

paradigm of expected utility:

\[ U(\hat{X}) = \mathbb{E}_\rho \left[ \sum \omega \text{d}\hat{X}(\omega, \cdot) \right] \]

independence axiom:

\[ \hat{X} \succ \hat{Y} \Rightarrow \alpha \hat{X} + (1-\alpha) \hat{Z} \succ \alpha \hat{Y} + (1-\alpha) \hat{Z} \]

for any \( \alpha \in (0, 1) \), any \( \hat{Z} \)
However:

"Paradox x\(a\)"

= deviations from classical "rationality"

(Allais, Ellsberg, ..., Kahneman, Tversky, ...)

\(\sim\) "Knightian uncertainty"
present state of the art/wisdom
beyond "expected utility":

\[ U(x) = -g(c(x)) \]

"upside"

"downside"

\[ u = \text{utility function} \]

\[ g = \text{"convex risk measure"} \]

\[ \iff \]

Maccheroni
Rustichini
Marinacci
(Econometrica 2006)

relaxation of
"rationality" axioms

in particular of the
"independence axiom"
Special cases:

- \( g = \text{a "coherent risk measure"} \)

Gilboa- Schmeidler

- \( g = -E_p \)

von Neumann- Morgenstern
Savage
Plan:

- Short introduction to convex/coherent risk measures

Artzner, Delbaen, Eber, Heath, Mathematical Finance (1999)

↓

- Structure of robust preferences
  (Hans Föllmer / Stefan Weber)

- Robust portfolio choice
  (Stefan Weber / Alex Schied)