The classical $N$–body problem consists of analyzing the state of a system of $N$ interacting bodies, where the force exerted on each body in the system results from its interaction with all the other bodies in the system. There are typically $O(N^2)$ operations necessary to compute the forces involved. In solving Singular Surface Integrals (Boundary Element Method, Method of Moments), we encounter a similar problem: Discretizing the underlying surface integrals, and, thus, introducing $N$ unknowns, we generally need to compute a large number of interactions requiring $O(N^2)$ operations. Thereby, the interaction coefficients depend on the relative Euclidean distance between the interacting elements in a rather complicated form. Hierarchical tree-structured algorithms have been developed to reduce the complexity of computations. The key to the majority of standard fast algorithms is the availability of potential functions in factorized form, which are obtained by applying the so-called Addition Theorem. A question arises as how to proceed if a factorized form is not available, or cannot be easily constructed. It is the prime purpose of this work to present a general recipe for factorizing potential-and field distribution functions without utilizing the Addition Theorem.