NUMERICAL METHODS FOR INTERACTIVE MULTIPLE CLASS IMAGE SEGMENTATION PROBLEMS

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Abstract. In this paper, we consider a bilaterally constrained optimization model arising from the semi-supervised multiple class image segmentation problem. We prove that the solution of the corresponding unconstrained problem satisfies a discrete maximum principle. This implies that the bilateral constraints are satisfied automatically and that the solution is unique. The structures of coefficient matrices arising from the optimality conditions of the segmentation problem are different for different input images, we still show that they are $M$-matrices in general. Therefore we study several numerical methods for solving such linear systems and demonstrate that domain decomposition with block relaxation methods are quite effective and outperform other tested methods. We also carry out a numerical study of condition numbers on the effect of boundary conditions on the optimization problems which provides some insights into the specification of boundary conditions as an input knowledge in the learning context.

Key words. Image segmentation, discrete maximum principle, domain decomposition, $M$-matrix, condition numbers, boundary conditions

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