Refined Long-Time Asymptotics for Some Polymeric Fluid Flow Models

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Abstract. We consider a polymeric fluid model, consisting of the incompressible Navier-Stokes equations coupled to a non-symmetric Fokker-Planck equation. First, steady states and exponential convergence to them in relative entropy are proved for the linear Fokker-Planck equation in the Hookean case. The FENE model is also addressed proving the existence of stationary states and the convergence towards them in suitable weighted norms. Then, using the “entropy method” exponential convergence to the steady state is established for the coupled model in the Hookean case under some smallness assumption. The results continue and expand the analysis of [19] in both the Hookean and the FENE models.

Key words. entropy method, relative entropy, Fokker-Planck equations, large time behavior, exponential decay rate, polymeric flow, dumbbell model.

AMS subject classifications. 35K15, 35Q30, 35B40; 76T20.

1. Introduction

We consider a coupled microscopic-macroscopic model for a dilute solution of polymers in a homogeneous fluid. The incompressible Navier-Stokes equations for the macroscopic flow shall be coupled via the stress tensor to a microscopic model for the polymer chains distributed within the fluid (cf. [9, 10, 14, 28] for the physical background of such models). Let us briefly review the coupled model for the polymer distribution within a macroscopic flow. After putting the system in non-dimensional form and setting all remaining dimensionless parameters equal to one for notational simplicity, it reads as follows:

\[
\begin{align*}
\frac{\partial u}{\partial t} + (u(t,x) \cdot \nabla x)u(t,x) &= \Delta x u(t,x) - \nabla x p(t,x) + \text{div} x \tau(t,x), \\
\text{div} x u &= 0,
\end{align*}
\]

(1.1)

\[\tau(t,x) = \int_{\mathbb{R}^d} d(X \otimes \nabla X) \Pi(X) \psi(t,x,X) dX,
\]

(1.3)

where \(u(t,x), x \in \Omega \subset \mathbb{R}^d, d \geq 2\) is the velocity field of the fluid, \(p(t,x)\) the pressure, and \(\tau(t,x)\) the stress tensor. This system is coupled through Eq. (1.3) to the following microscopic model for the polymer evolution. Here, the polymers are modeled as dumbbells of length and orientation given by the vector \(X \in \mathbb{R}^d\). The Fokker-Planck–type evolution equation for \(\psi(t,x,X)\), the probability density of dumbbells w.r.t. the

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