Heisenberg Operators of a Dirac Particle Interacting with the Quantum Radiation Field

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Abstract

We consider a quantum system of a Dirac particle interacting with the quantum radiation field, where the Dirac particle is in a $4 \times 4$-Hermitian matrix-valued potential $V$. Under the assumption that the total Hamiltonian $H_V$ is essentially self-adjoint (we denote its closure by $\overline{H}_V$), we investigate properties of the Heisenberg operator $x_j(t) := e^{it\overline{H}_V} x_j e^{-it\overline{H}_V}$ $(j = 1, 2, 3)$ of the $j$-th position operator of the Dirac particle at time $t \in \mathbb{R}$ and its strong derivative $dx_j(t)/dt$ (the $j$-th velocity operator), where $x_j$ is the multiplication operator by the $j$-th coordinate variable $x_j$ (the $j$-th position operator at time $t = 0$). We prove that $D(x_j)$, the domain of the position operator $x_j$, is invariant under the action of the unitary operator $e^{-it\overline{H}_V}$ for all $t \in \mathbb{R}$ and establish a mathematically rigorous formula for $x_j(t)$. Moreover, we derive asymptotic expansions of Heisenberg operators in the coupling constant $q \in \mathbb{R}$ (the electric charge of the Dirac particle).

Keywords: Dirac-Maxwell operator; Dirac operator; Dirac particle; Heisenberg operator; position operator; quantum radiation filed; velocity operator; Zitterbewegung