GEOMETRICAL VERSIONS OF IMPROVED BEREZIN-LI-YAU INEQUALITIES

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Abstract. We study the eigenvalues of the Dirichlet Laplace operator on an arbitrary bounded, open set in $\mathbb{R}^d$, $d \geq 2$. In particular, we derive upper bounds on Riesz means of order $\sigma \geq 3/2$, that improve the sharp Berezin inequality by a negative second term. This remainder term depends on geometric properties of the boundary of the set and reflects the correct order of growth in the semi-classical limit.

Under certain geometric conditions these results imply new lower bounds on individual eigenvalues, which improve the Li-Yau inequality.

2000 Mathematics Subject Classification. Primary 35P15; Secondary 47A75.

The work was done partially while the third author was visiting the Institute for Mathematical Sciences, National University of Singapore in 2010. The visit was supported by the Institute.