OPTIMAL ERROR ESTIMATES OF FINITE DIFFERENCE METHODS FOR THE GROSS-PITAEVSKII EQUATION WITH ANGULAR MOMENTUM ROTATION

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ABSTRACT. We analyze finite difference methods for the Gross-Pitaevskii equation with an angular momentum rotation term in two and three dimensions and obtain the optimal convergence rate, for the conservative Crank-Nicolson finite difference (CNFD) method and semi-implicit finite difference (SIFD) method, at the order of $O(h^2 + \tau^2)$ in the $L^2$-norm and discrete $H^1$-norm with time step $\tau$ and mesh size $h$. Besides the standard techniques of the energy method, the key technique in the analysis for the SIFD method is to use the mathematical induction, and resp., for the CNFD method is to obtain a priori bound of the numerical solution in the $L^\infty$-norm by using the inverse inequality and the $L^1$-norm error estimate. In addition, for the SIFD method, we also derive error bounds on the errors between the mass and energy in the discretized level and their corresponding continuous counterparts, respectively, which are at the same order of the convergence rate as that of the numerical solution itself. Finally, numerical results are reported to confirm our error estimates of the numerical methods.

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