Gibbs-Preserving Maps outperform Thermal Operations in the quantum regime

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In this brief note, we compare two frameworks for characterizing possible operations in quantum thermodynamics. One framework considers Thermal Operations—unitaries which conserve energy. The other framework considers all maps which preserve the Gibbs state at a given temperature. Thermal Operations preserve the Gibbs state; hence a natural question which arises is whether the two frameworks are equivalent. Classically, this is true—Gibbs-Preserving Maps are no more powerful than Thermal Operations. Here, we show that this no longer holds in the quantum regime: a Gibbs-Preserving Map can generate coherent superpositions of energy levels while Thermal Operations cannot. This gap has an impact on clarifying a mathematical framework for quantum thermodynamics.

In recent approaches to thermodynamics [1–13], one specifies a set of state transformations which an experimenter is allowed to perform “for free”, i.e. at no work cost—such a framework is called a resource theory. Among the various mathematical frameworks proposed to model thermodynamical operations, two have proven particularly useful, namely the resource theory of Thermal Operations and the Gibbs-Preserving Maps. Classically, these two frameworks are equivalent. If a transition between initial and final states block diagonal in their energy eigenbasis is possible by Gibbs-Preserving Maps, then it is also possible via Thermal Operations [7]. One might suppose that this equivalence holds for arbitrary quantum states. In this short note, we show that this is not the case: Gibbs-Preserving Maps can perform transitions which Thermal Operations are incapable of.

Thermal Operations.—The resource theory of Thermal Operations has been extensively exploited to understand thermodynamics at the quantum level [4, 7, 14–16]. One is allowed to perform any arbitrary joint unitary operation, on a system and a bath, which conserves the total energy on any state consistent with the initial density matrix of the input. Thermal Operations also include bringing in arbitrary systems which are in the Gibbs state (with arbitrary Hamiltonians). Finally, Thermal Operations allow subsystems to be discarded for free, regardless of their state. Observe that Thermal Operations cannot change the Gibbs state into any other state [4, 7, 14]. What’s more, the Gibbs state is the only state which has this property [13]. Gibbs states are thus the only state which can be allowed for free—if any other state were allowed, arbitrary state transformations would be possible.

Crucially, Thermal Operations are not capable of generating coherent superpositions of energy levels: a Thermal Operation must, by definition, commute with the total Hamiltonian, and thus cannot generate such a superposition starting from an energy eigenstate.

Gibbs-Preserving Maps.—In the framework of Gibbs-Preserving Maps, one allows to carry out any completely positive, trace-preserving map on a system which preserves the Gibbs state (or “Gibbs-Preserving Map”, for short). These maps are a natural quantum-mechanical generalization of the stochastic matrices used to characterize the so-called d-majorization or mixing character [17–21]. In any reasonable thermodynamical framework, a map that does not preserve the Gibbs state must cost work; this fact makes Gibbs-Preserving Maps a conservative choice of framework for proving fundamental limits.

Since a Thermal Operation preserves the Gibbs state, the state transformations possible with Thermal Operations are necessarily included in those achievable with Gibbs-Preserving Maps. Is the converse true? It is for states which are block diagonal in their energy eigenbasis. This can be seen as follows. A necessary and sufficient condition for transitions via Thermal Operations is thermo-majorization [7], a partial order which is a generalization of majorization [20, 22–24]. More precisely, transformations are completely characterized in terms of thermo-majorization of the initial and final states’ spectrum with respect to the Gibbs state. Classic results about majorization ensure the existence of a stochastic matrix mapping the vector of eigenvalues of the initial state to the final state’s eigenvalues, while preserving the Gibbs state’s spectrum. Written out in full as a channel on quantum states, it is a Gibbs-Preserving Map [25].

We now address the question of whether Gibbs-Preserving Maps are strictly more powerful than Thermal Operations, on arbitrary input states. We show that this is the case, by exhibiting an example of a Gibbs-Preserving Map that performs a transformation forbidden by Thermal Operations.

The Example.—Consider a two-level system with an energy gap $\Delta E$. We denote the ground state by $|0\rangle$ and the excited state by $|1\rangle$. Consider now the transformation:

$$|1\rangle \rightarrow \rho ,$$

(1)