A new approach to Poisson approximation and applications

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Abstract

The main purpose of this article is to introduce a new approach to Poisson approximation. Some bounds in Poisson approximation for probability distributions of a wide class of various arrays of row-wise independent discrete random variables are established via a probability distance. Some analogous results related to random sums in Poisson approximation are also considered.

Keywords: Poisson approximation, Random sums, Le Cam’s inequality, Trotter’s operator, Poisson-binomial random variable, Geometric random variable, Negative-binomial random variable, Integer-valued random variable, Probability distance

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1 Introduction

Let $X_{n,1}, X_{n,2}, \ldots$ be a triangular array of row-wise independent Bernoulli distributed random variables with success probabilities $P(X_{n,j} = 1) = 1 - P(X_{n,j} = 0) = p_{n,j} \in (0,1), j = 1, 2, \ldots, n; n \geq 1$, such that $\hat{p}_n := \max_{1 \leq j \leq n} p_{n,j} \to 0$ as $n \to \infty$. Set $S_n = \sum_{j=1}^{n} X_{n,j}$ and $\lambda_n = E(S_n) = \sum_{j=1}^{n} p_{n,j}$. Suppose that $\lim_{n \to \infty} \lambda_n = \lambda, \quad (0 < \lambda < +\infty)$. We will denote by $Z_\lambda$ the Poisson random variable with mean $\lambda$. It has long been known that the probability distributions of $S_n, n \geq 1$ are usually approximated by the distribution of $Z_\lambda$. More specifically, under above assumptions on $\hat{p}_n$ and $\lambda$, the Poisson approximation theorem states that

$$S_n \overset{d}{\to} Z_\lambda \quad \text{as} \quad n \to \infty. \quad (1)$$

(For other surveys, see [?], [?], [?], [?], [?], [?]). Note that, and from now on, the notation $\overset{d}{\to}$ means the convergence in distribution. Moreover, using the method of convolution operators, Le Cam ([?], 1960) obtained the remarkable inequality

$$\sum_{k=0}^{\infty} | P(S_n = k) - P(Z_\lambda = k) | \leq 2 \sum_{j=1}^{n} p_{n,j}^2. \quad (2)$$

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